RESEARCH ARTICLE | JUNE 05 2023

Inhibition and activation of interactions in networked weak prisoner's dilemma

Yichao Yao ; Ziyan Zeng ; Bin Pi ; Minyu Feng

Chaos 33, 063124 (2023)
https://doi.org/10.1063/5.0146999

Chaos ARTICLE scitation.org/journal/cha

Inhibition and activation of interactions in networked weak prisoner's dilemma

Cite as: Chaos **33**, 063124 (2023); doi: 10.1063/5.0146999
Submitted: 18 February 2023 · Accepted: 18 May 2023 ·
Published Online: 5 June 2023

View Online Export Citation CrossMark

Yichao Yao, Ziyan Zeng, Bin Pi, and Minyu Feng[a)]

AFFILIATIONS

College of Artificial Intelligence, Southwest University, Chongqing 400715, China

[a)]**Author to whom correspondence should be addressed:** myfeng@swu.edu.cn

ABSTRACT

In the framework of the coevolution dynamics of the weak prisoner's dilemma, inspired by prior empirical research, we present a coevolutionary model with local network dynamics in a static network framework. Viewing the edges of the network as social interactions between individuals, when individuals play the weak prisoner's dilemma game, they accumulate both payoffs and social interaction willingness based on a payoff matrix of the social interaction willingness we constructed. The edges are then inhibiting or activating based on the social interaction willingness of the two individuals, and individuals only interact with others through activated edges, resulting in local network dynamics in a static network framework. Individuals who receive more cooperation will be more likely to activate the edges around them, meaning they will participate in more social interactions. Conversely, individuals who receive more defects will do the opposite. Specifically, we investigate the evolutionary dynamics of cooperation under different levels of sensitivity to social interaction willingness and the temptation to defect. Through the simulation, we find that sparse cooperator clusters can expand greatly when social interaction sensitivity and temptation to defect are low. In contrast, dense cooperator clusters form rapidly in a high social interaction sensitivity, which protects the cooperation from high temptation.

In order to preserve the characteristics of the network during the coevolution process, we employ a coevolution model that transforms global network dynamics into local dynamics. Specifically, individuals acquire social interaction willingness based on the weak prisoner's game outcomes and subsequently activate or inhibit their surrounding edges based on their respective social interaction willingness. The social interaction willingness of individuals is determined by a social interaction payoff matrix that we construct, wherein we carefully examine the necessary conditions that the matrix elements must satisfy to ensure that individual behavior aligns with reality. To investigate the effects of individual social interaction intention and the temptation to defect, we conduct simulations that explore the dynamics of local networks around various individuals, including the protective, blocking, and promoting mechanisms of cooperation under varying levels of social interaction sensitivity.

I. INTRODUCTION

Cooperative behaviors can be found in a wide range of areas, from interactions between nations to the social lives of individuals and even to interactions between cells, which is the foundation of social development and prosperity. Therefore, the research on cooperation has paramount importance in identifying which conditions help or which ones block the spreading of altruistic behavior in a complex population,[1,2] and researchers from a variety of disciplines have been attracted to study the evolution of cooperation.[3] In the research on the evolution of cooperation, the evolutionary game theory provides a practical research framework for exploring the potential mechanisms for cooperation dilemmas.[4–6] In addition, researchers have proposed various evolutionary cooperation mechanisms that have a basis in reality, such as Nowak's five rules in 2006,[7] including the network reciprocity model, which is of great significance to the research on the evolution of cooperation.[5,8] Meanwhile, complex networks have been extensively studied and developed rapidly in the past few decades as an essential method for modeling complex systems. Many networked game models have been proposed and combined with evolutionary game theory and widely used to study cooperative evolution based on the hypothesis of group or pair interaction. Currently, the prisoner's dilemma game (PDG) and snowdrift game (SDG) have been studied extensively,[9] so as the public goods game (PGG).[10] The research results have shown that the network structure is a powerful cooperation-promoting mechanism in the prisoner's dilemma,[11] but this is not the case in the snowdrift dilemma. Structure, surprisingly, decreases the frequency of cooperators relative to well-mixed populations.[9,12] Instead of large, compact clusters common in spatial prisoner's dilemma games, clusters in spatial snowdrift dilemma games are small and filament-like[9] due to the fact that two interacting snowdrift players should adopt opposite strategies to one another. In addition to the node-decision model, there is also a link-decision model,[13] which has the interaction diversity based on edge strategy that has been demonstrated to effectively enhance cooperation.[14]

Network reciprocity indicates that cooperators will resist the invasion of defectors by forming spatial clusters in networks, and the existence of a social structure may significantly affect the interaction mode between individuals, thus creating an environment in which cooperation can emerge and be maintained.[8,15] Nowak and May initially observed that repeated games in a square grid result in spatial chaos.[16] In structured populations, cooperators can expand by forming clusters, and despite being exploited by defectors at the boundaries of the clusters, they can still benefit from cooperation.[17] Afterward, systematic Monte Carlo simulations and generalized mean-field techniques are introduced into the structured prisoner's game,[18] and Santos and Pacheco demonstrated how important interactive structures are to the evolution of cooperation.[19] These findings have led to further research on whether cooperators can survive or even thrive in different types of network structures; random and scale-free networks have become a focus of research,[11,20,21] and evolutionary games on the small-world networks have been just well-analyzed.[22,23] These studies suggest that the greater heterogeneity of a network can provide optimal conditions for the evolution of cooperation by ensuring that cooperation clusters are less affected by defection at their boundaries.[24] The strong promoting effect of cooperation by scale-free networks has successfully garnered much attention in this field, but it was subsequently shown to lack robustness in relation to the assumptions of theoretical models.[25] Hereinbefore, most of the above studies only take individuals as the object of evolution, but in reality, the environment in which individuals live will also change constantly. Individuals in the game may be able to change the environment according to his/her own will, that is, to change the topological nature of the interaction structure between individuals. That is realized in the coevolution of network structure and individual strategy in the network. As a natural upgrade of static game theory on the network, coevolution rules add the changes in the network structure to the evolution process, which creates more possibilities for the evolutionary game on the networks.[26,27] The adaptability of a spatial structure has brought more space for individuals to individuate their behaviors. Initially, coevolutionary rules affecting the interactions between players were proposed by Zimmermann and Ebel *et al.*[28,29] Subsequently, Pacheco *et al.* designed more exquisite coevolution models and studied the impact of time-scale separation of network structure and individual evolution on cooperation.[26,30] The importance of this time-scale separation to the evolution of cooperation was further demonstrated by Rand *et al.* through human volunteer experiments.[31] Research on the adaptive network has also achieved rich results in recent years. So far, Szolnoki *et al.* have developed a model for making new connections on a lattice network, demonstrating the promoting effect of network dynamics on cooperation in structured prisoner's dilemma.[32] Subsequently, Szolnoki *et al.* created a coevolving model for making and deleting connections on the random network and observed that cooperation can still prevail even under the high temptation to defect in such coevolving models;[33] furthermore, they studied the emergence of multilevel selection in the prisoner's dilemma game on coevolving random networks, which despite the sustained random topology of the evolving network, maintains cooperation across the whole span of defection temptation values.[34] These research studies prove the importance of network dynamics for cooperation in the structured prisoner's dilemma from multiple perspectives, up to now, the changing environment was already the subject of many works,[35] and environmental feedback is shown to have a significant effect on cooperation.[36] More recently, reinforcement learning was discovered to facilitate an optimal interaction intensity for cooperation.[37] On the other side, network adaptation not only affects cooperation but also provides some consideration for the formation of network structure in actual society. For example, Li *et al.* modeled scale-free networks through evolutionary games on adaptive networks.[38] Holger *et al.* and Hiroki *et al.* obtained spontaneous emergence of network complexity from the adaptive evolution of networks.[29,39] In addition to the research on evolutionary dynamics theory, many scholars have made a lot of exploration on the emergence and maintenance of cooperative behavior through behavioral experiments,[40] such as the study of cognitive bias and communicating sentiment.[41]

Previous research on network dynamics has often focused on rewriting the edges in the network.[27] This approach assumes that individuals can select their neighbors in various ways and interact with all of them in each iteration. It also presupposes that the network structure can easily change with evolution. However, studies on evolutionary games in dynamic networks have shown that complex features appear and disappear on the interactive network as evolution progresses. In reality, the topological nature of the interactive network changes slowly. While we may switch departments or make new friends, it is unlikely that our social relationship network will easily alter its scale-free or small-world characteristics. Furthermore, even if social relations remain unchanged, our interactions with others may still vary. Over time, a person's willingness and frequency of interaction with others may change, and the intensity of social interaction may differ from person to person.[42,43]

To address the aforementioned issues, we propose a novel dynamic network model in which edges cannot be rewritten but can be activated or inhibited based on the individuals they connect with. Our approach differs from previous works in that the network for gaming will remain true to the original network, thus ensuring a relatively stable network structure and heterogeneous interactions between individuals. Furthermore, unlike models with varying weights, all individuals in our model will treat all neighbors equally, and the propagation of strategies will not be affected by local fitness levels.[44] In our proposed model, each player located on the node of the network accumulates social interaction intention A in the game, which affects the activation and suppression of edges. Individuals who encounter more cooperators in the game will accumulate more social interaction intention, and edges that connect individuals with higher interaction intention are more likely to be activated. On this basis, we present the social interaction intention payoff matrix and the sensitivity of individual social interaction intention to game situation r_a. The player plays the weak prisoner's dilemma game with the immediate neighbors on square-lattice networks and small-world networks. Through the simulations, we observe rich variations in the evolution of cooperation and the local network structure of heterogeneous individuals. The results of our simulations demonstrate that the cooperation can be promoted by the network's dynamic nature and the sensitivity of individual social interaction intention to game situations in a wide range.

The rest of this paper is organized as follows. In Sec. II, we introduce the evolutionary game model and the dynamic network model based on social interaction intention. Section III presents the simulation results obtained by different initial networks and parameter settings and then explains their internal laws and mechanisms. In Sec. IV, we provide conclusions and outlooks.

II. MODELING

This section introduces the model of evolutionary games employed in our study. In the network evolutionary game model, the nodes in the network represent individuals, while the edges signify social interactions between them. The individuals connected by edges engage in games with each other, apply their strategies, obtain payoffs, and subsequently update their strategies based on their payoffs. Drawing inspiration from the varying interaction frequencies and intentions observed among individuals in real society, our model additionally incorporates the activation and inhibition of connecting edges in the network to simulate changes in the intensity of social interactions among individuals. To distinguish our study from those involving dynamic weights, we note that while individuals in our model exhibit heterogeneous social interactions with their neighbors, which affects the ease of strategy communication based on their willingness to engage socially, they treat all neighbors equally when accumulating income and learning strategies. Furthermore, the propagation difficulty of an individual's strategy is not directly impacted by the fitness of the neighborhood, but rather by changes in the degree.[44] In the subsequent sections, we will introduce the game model, strategy evolution rules, and edge activation rules utilized in our study.

A. Symmetric paired games and strategy evolution

In our model, we adopt a symmetrical two-person, two-strategy game, in which individuals can choose to cooperate or defect. The strategy space and payoff matrix are identical for both players in the game. Specifically, in each elementary game, players receive a reward of R for mutual cooperation and a punishment of P for mutual defection. A defector receives the temptation T from a cooperator, while the cooperator receives the sucker's payoff S. The resulting payoff matrix U can be expressed as

$$U = \begin{pmatrix} R & S \\ T & P \end{pmatrix} \tag{1}$$

for an individual; the first and second rows represent the individual's choices to cooperate or defect, respectively, while the first and second columns represent their opponent's corresponding choices, and the matrix element represents the payoff. The relative values of the four parameters determine the type of game model. For instance, the prisoner's dilemma model is obtained when $2R > T + S$ and $T > R > P > S$, while the snowdrift game model is obtained when $2R > T + S$ and $T > R > S > P$. In our proposed model, we utilize the weak prisoner's dilemma game, wherein the payoff for an individual choosing to defect is never less than that for choosing to cooperate, but mutual defection results in a lower return than mutual cooperation. This represents a typical social dilemma.

In each iteration, each individual plays a game with all its neighbors who have activated edges connecting to them. After all the players have acquired their payoffs, each individual will have the opportunity to update their strategy once, e.g., an individual named x obtains payoff u_x in one iteration, we randomly select one of its neighbors, y, who receives the payoff u_y at the same time, the probability that x imitates its neighbor y's strategy is given by the Fermi–Dirac function,[18,24,45]

$$P(s_x \leftarrow s_y) = \frac{1}{1 + \exp{(u_x - u_y)/\kappa}}, \tag{2}$$

where s_i represents the strategy of i. Specifically, if $u_y > u_x$, where u_i denotes the payoff that individual i received in the previous turn, then the probability that individual x imitates the strategy of y is positively correlated with the difference between u_y and u_x. The noise factor κ modulates the impact of the payoff difference on the strategy updating process. When $\kappa \to 0$, strategy learning will be clear and definite, and if $\kappa \to \infty$, strategy learning will become completely random. If individual x does not update its strategy after playing with y in a round, it will maintain its current strategy.

B. Activation of edges

In each iteration of the evolution, the edges on the network are activated according to the individual's willingness to interact socially, and the individuals play games only through the activated edges. The individual's willingness to interact should be related to the quality of the social environment, specifically, the proportion of cooperative and defection encounters. If an individual encounters cooperators more frequently, it implies a friendly social environment, which promotes the individual's social interaction intention. Conversely, if an individual encounters defectors more frequently, it indicates a worse environment, and the presence of defectors may inhibit the individual's social interaction intention. Therefore, we can define the income matrix U_a of social interaction willingness by imitating the payoff matrix U of a two-person two-strategy game to quantify the accumulated social interaction intention of individuals in their social interactions and denote it as

$$U_a = \begin{pmatrix} R_a & S_a \\ T_a & P_a \end{pmatrix}. \tag{3}$$

As illustrated in this matrix, in each iteration, mutual cooperation results in the accumulation of social interaction intention R_a for both parties, while mutual defection results in the accumulation of social interaction intention P_a for both parties. When a cooperator is defected, its social interaction intention will increase S_a, while the social interaction intention of a defector will increase T_a. Since cooperation should enhance social interaction intention, while defection should inhibit it, the following inequalities should hold

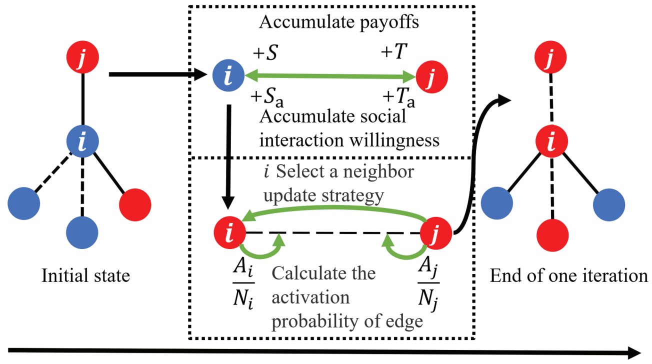


FIG. 1. An example for the evolution. The steps are included in a round of iteration, in which the red dot represents the defector and the blue dot represents the cooperator. The solid line and the dotted line represent activated and inhibited edge, respectively. The green edges and arrows represent the possible interaction between individuals and edges. Here, we take the individual i as an example, through gameplay with activated edges, i accumulates payoffs (u_i) and social interaction intentions (A_i). Following this, i updates the strategy and distributes the social interaction intentions evenly to the edges, thereby enabling the activation or inhibition of the edges in the next iteration.

true: $2R_a > T_a + S_a$ and $R_a > P_a$. As for the inequality between $2P_a$ and $S_a + T_a$, it should be indeterminate since we cannot ascertain whether the defection of the cooperator or mutual defection has more negative effects on social interaction.

Next, we must establish a correlation between the social interaction intention of individuals and the activation and inhibition of edges. Based on the definition above, the accumulation of social interaction willingness is closely related to the degree of the individual, similar to the accumulation of payoff in the game. A larger degree of an individual results in a higher upper limit of its interaction willingness accumulation. In order to average the influence of degree heterogeneity while avoiding discrimination against specific strategy individuals, we can equally distribute the accumulated social interaction intention of each individual to all the edges it possesses. Then, the probability of edge activation in the next time step is determined by the sum of the two social interaction intention values it received. For example, the activation probability of an edge between individual i and individual j is given by

$$P = \frac{A_i}{|N_i|} + \frac{A_j}{|N_j|}, \tag{4}$$

where $|N_i|$ and $|N_j|$ are the number of neighbors of i and j, and A_i and A_j are the accumulated social interaction intention of i and j. As shown in Fig. 1, each individual plays the game once with all the activated edges with the neighbors to accumulate their social interaction intention [Eq. (3)], and this is identified with the accumulation of payoffs.

In addition, we use C to represent the cooperator and D to represent the defector. When considering the total amount of interaction tendency provided by any pair of relations, the values of C–C, D–D, and C–D relations are $2R_a$, $2P_a$, and $S_a + T_a$, respectively. For a pair of C–C relations, the expected value of the number of edges activated by $2R_a$ should be greater than 1 to reflect its promotion effect on social interaction. In contrast, for the inhibition of D–D and C–D relations on social interaction, the expected number of edges that $2P_a$ and $S_a + T_a$ can activate should be less than 1. To simplify the model and ensure its directness, we take the value of the social interaction intention assigned to an edge as the probability of activating the edge. The parameters should, thus, satisfy $2R_a > 1$, $2P_a < 1$, and $S_a + T_a < 1$. When a relationship provides both individuals with an interaction willingness of 0.5, the value they get from that relationship is, thus, expected to activate one edge in the next round, which keeps the number of edges constant. Therefore, it will be reasonable to set the baseline value at 0.5 for the social interaction willingness provided by a relationship. As the interaction willingness provided by a relationship exceeds 0.5, it promotes an individual's social interaction, and the individual is expected to activate more edges in the next round. Conversely, if the interaction willingness provided is less than 0.5, it inhibits the individual's next round of social interaction, and the individual contributes less to the activation of the edges. Moreover, when a defector is cooperated with by a cooperator, its social interaction intention should be enhanced for the C–D relationship. However, as the C–D relationship is ultimately oppressive, we assume that its promotion of the defector's social interaction intention is less than that of the C–C relationship, i.e., $R_a > T_a$. Therefore, we conclude that the elements in the social interaction intention matrix must satisfy the following inequality conditions:

$$\begin{cases} R_a > T_a > 0.5, \\ S_a + T_a < 1, \\ P_a < 0.5. \end{cases} \tag{5}$$

To ensure that the accumulation of individuals' willingness to social interaction conforms to the general social pattern of reality, we need to satisfy Eq. (5) for the elements of Eq. (3). Specifically, cooperation should have the ability to promote social interactions while defection should inhibit social interactions.

Figure 1 shows the actions to be performed in one iteration. At the start of the iteration, individuals use their strategies to play games with their activated edges, accumulating their social interaction willingness and profits concurrently. Subsequently, the strategy and activated edges are updated through Eqs. (2) and (4), concluding the current round of evolution.

III. SIMULATION AND RESULTS

In this section, we present the simulation method and the results to demonstrate the influence of some parameters on cooperative evolution and the difference in the evolution under different parameter combinations.

A. Method

For the game model, we consider the weak prisoner's dilemma with the payoff matrix

$$U = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}, \tag{6}$$

where $1 \leq b < 2$ is a regulable parameter, and it is not the typical PDG but reduces the four parameters in PDG to one,[16] which makes the experiment of an evolutionary game convenient to control, simulate, and display. For the payoff matrix of individuals' social interaction willingness, we adopt a similar approach to introducing a uniquely adjustable parameter. We introduce the parameter r_a to quantify the degree of promotion of cooperation and inhibition of defection in social interaction, thereby controlling the sensitivity of individuals' social interaction willingness to the environment. We assume that the interaction willingness provided by D–D relationships is zero, and after making assumptions about the coefficients of r_a in other parameters, the matrix takes the form

$$U_a = \begin{pmatrix} 0.5 + r_a & 0.5 - 1.5r_a \\ 0.5 + 0.5r_a & 0 \end{pmatrix}. \quad (7)$$

It should be noted that the coefficient of r_a may have an impact on the evolution, but for the sake of simplicity in simulations and presentations, we require a parameter to adjust and measure the influence tendency of different types of interaction on an individual's social interaction intention. Under the current setting, when $r_a \to 0$, only the D–D relationship inhibits social interaction, while C–C and D–D relationships have no impact. As r_a increases, C–C and C–D relationships will increasingly promote and inhibit social interaction, and in C–D relationships, both cooperators and defectors are affected to a greater extent. While all parameters must satisfy Eq. (5), this set of parameters is consistent with the category and requirements of our research. Therefore, in this paper, we will not discuss the influence caused by the coefficient of r_a, but simply set it to an acceptable value.

We conducted simulations on two types of networks: the square lattice with periodic boundary (SL) and the WS network proposed by Watts and Strogatz. The SL network consists of 40×40 nodes, while the WS network has 1500 nodes with an average degree of 4 and a random reconnection probability of 0.3. Each individual has the opportunity in each iteration to learn the strategy of a randomly selected neighbor with a probability given by Eq. (2), where κ is set to 0.1. At the initial state of the evolution, we assign each individual an equal probability of choosing to cooperate or defect as their initial strategy and randomly activate half of the edges in the network. To ensure the reliability of the results, we conducted each simulation more than ten times and averaged the results.

In the following part, we present the statistical characteristics of the evolution of cooperation and social interaction on the network under various parameter combinations.

B. Evolution of cooperation

In this subsection, we focus on the fraction of cooperation after the evolution reaches stability under different parameter sets and the evolution of cooperation over time. Figure 2 displays the variation of cooperation with the number of iterations under different parameter values. We observe that the evolution on SL networks reaches stability after about 150 iterations, while the evolution on the WS network reaches stability after approximately 200 iterations. In the early stage of evolution, there is a significant decline in cooperation, and the lowest point of cooperation decline differs with different parameters. As shown in Fig. 2, although the curve corresponding to $r_a = 0.2$ exhibits a lower lowest point of cooperation than the other curves, it gradually increases after reaching the nadir and eventually attains the highest fraction of cooperation. Conversely, the curves of r_a from 0.6 to 3 have a higher lowest point and reach stability faster than that of $r_a = 0.2$. For the parameter combinations displayed in this figure, a larger r_a indicates less time for the evolution to reach equilibrium and a lower final cooperation ratio.

For the evolution results for different parameter combinations, Fig. 3 illustrates the cooperation ratio attained by the system after evolution reaches stability, under varying defect temptation b and sensitivity r_a. Specifically, b is set within the range of 1–2, and r_a within 0–3, in both the SL and WS networks. We conduct 500 iterations for each simulation in Fig. 3, with results recorded after evolution reaches stability. Analysis of Fig. 3 reveals that when r_a is set to 0, even if b is set to 1, the fraction of cooperators on the network is exceedingly low, and gradually decreases with the increase of b. This phenomenon can be attributed to the variation of cooperation presented in Fig. 2 and the setup of Eq. (7). The cooperator ratio will have a period of rapid decline in all the parameter combinations in Fig. 2, as a result of our initial setting that the cooperator and defector in the network are evenly mixed at the beginning of evolution, allowing defectors to invade cooperation in large numbers, resulting in the rapid decline in the cooperation fraction in all cases. The further analysis focuses on the setting of Eq. (7), when r_a is set to 0, the intention of social interaction is not affected by C–C and C–D relations, while only D–D relations lead to a decline in the intention of social interaction for both individuals. In the absence of any mechanism promoting social interaction, the edges of the network gradually diminish. The proliferation of defectors in the early stages of evolution can sever almost all edges in the network, thereby impeding the spread of cooperation strategy. Consequently, isolated cooperators and defectors are left on the network, leading to the stasis in evolution depicted in Figs. 2 and 3.

We set the step size of r_a to 0.6 and included an additional result for $r_a = 0.2$ to show the evolution of the cooperation when r_a is small. As illustrated in Fig. 3, for any case where r_a is greater than 0, the final cooperation fraction on the network is better than that on a static network with the same b. Notably, the SL network exhibits a more pronounced improvement, suggesting that the heterogeneity of individuals' willingness to participate in social activities does promote cooperation. Among the selected parameter groups, a smaller r_a leads to a higher cooperation fraction when b is small (below 1.55 on the SL network and below 1.6 on the WS network), and $r_a = 0.2$ yields the highest cooperation fraction. However, when b surpasses a threshold, the fraction of cooperation obtained by a small r_a decreases rapidly with an increase of b and is lower than that obtained by other r_a settings; therefore, $r_a = 0.6$ achieves the best cooperation frequency in this situation. It can be seen that as the temptation for defectors increases, individuals' willingness to participate in social interaction needs to be more sensitive to guide the cooperation on the network to the optimum level. These details are apparent in the phase transition of the parameter combinations shown in Fig. 4, where a smaller r_a is more effective in promoting cooperation on the SL network, while on the WS network, the cooperation peak shifts to higher r_a more distinctly when b is larger. Additionally, the change of the cooperation fraction on the WS network is smoother than that on the SL network in the parameter

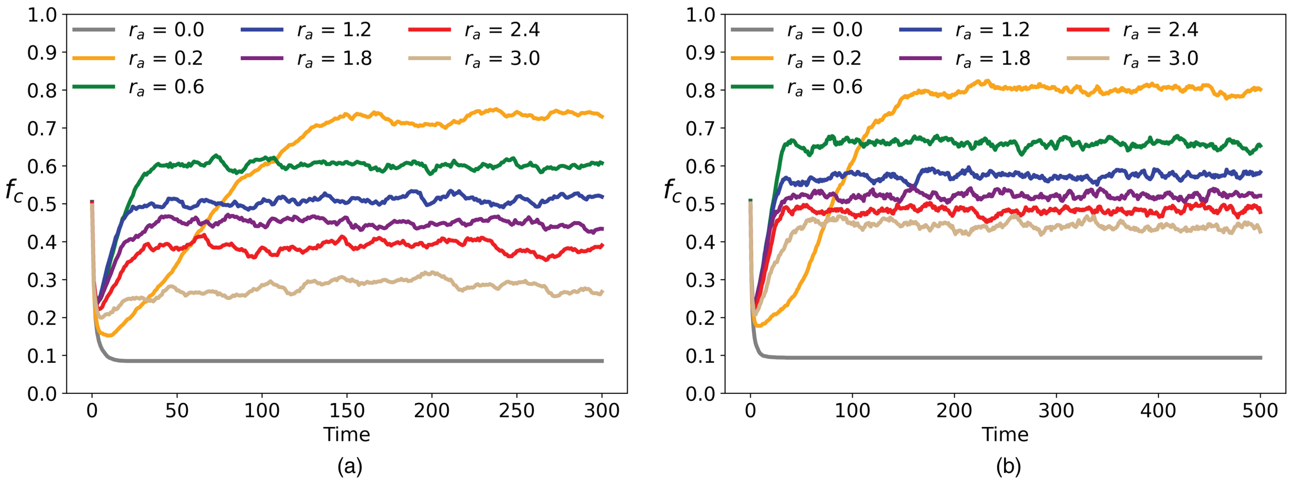


FIG. 2. Time evolution of f_c obtained for different r_a. The changes of f_c with the number of evolution iterations T, as obtained on SL and WS, where the sensitivity of the individual's social interaction intention is determined by r_a. The temptation to defect b is set to 1.45 for SL networks and 1.50 for WS networks. The gray, yellow, blue, purple, red, and brown curves represent the r_a values of 0, 0.2, 0.6, 1.2, 1.8, 2.4, and 3.0, respectively. The cooperation thrives best at a small r_a, and evolution stabilizes at around 200 steps.

space, and the evolution results on the SL network undergo a more considerable change in specific parameter adjustments.

C. Evolution of individual social interactions

To investigate the impact of heterogeneous social interaction intention on cooperation, we conducted further simulations to observe the change in social interaction intensity of different individuals during the evolution. Based on the evolutionary results obtained in the previous subsection, we selected three values of r_a—0.2, 0.6, and 3.0—to represent low, moderate, and high social interaction sensitivity, respectively. We then set b to 1.45 in the SL network and 1.5 in the WS network and observed the social interaction intention of individuals in different environments in these two networks. In each iteration of the simulation, we focus on those individuals whose local cooperation frequency f_c is within a certain range and record the average activation ratio of their surrounding edges in the next iteration. The results are shown in Fig. 5, the y axis represents the ratio of cooperators encountered by individuals in all their interactions in one round, that is, the neighborhood cooperation

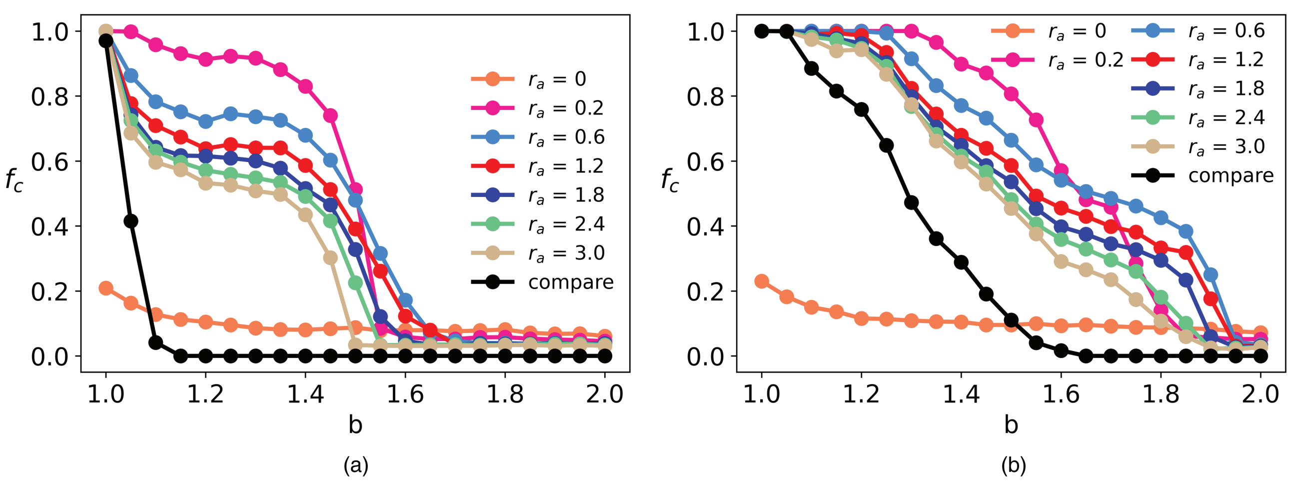


FIG. 3. Promotion of cooperation in dependence on b for r_a from 0 to 3.0. Fraction of cooperators f_c in dependence on different temptations to defect b, as obtained on SL and WS, where the sensitivity of the individual's social interaction intention is determined by r_a. The orange, pink, blue red, purple, green, and brown curves represent the r_a values of 0, 0.2, 0.6, 1.2, 1.8, 2.4, and 3.0 respectively, while the black curves represent the evolution results on the static network with the same parameters for comparison. When b is small, cooperation reaches its optimal level at r_a close to 0.2, while when b is large, cooperation reaches its optimal level at r_a close to 0.6.

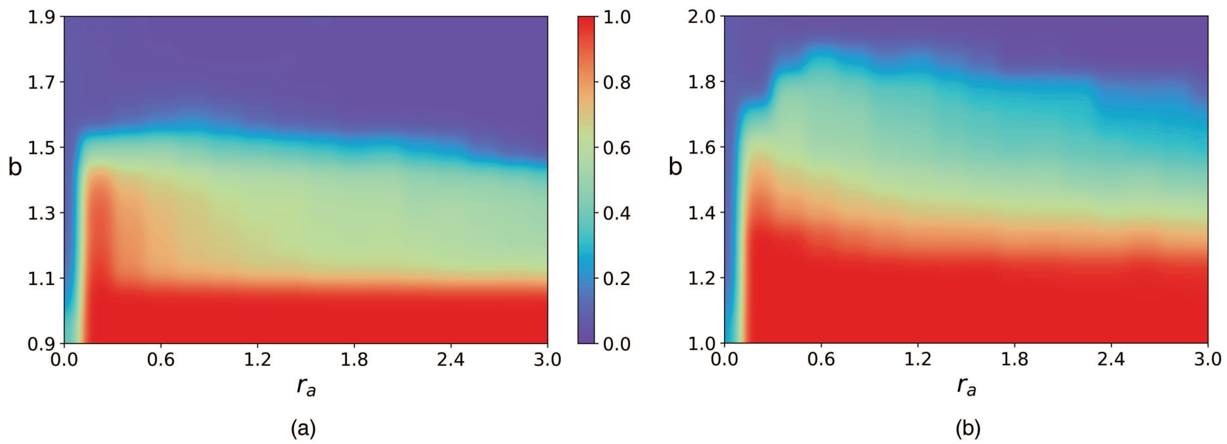


FIG. 4. Fraction of cooperation for different parameter pairs (b, r_a). The heat map shows the cooperation frequency obtained by different defectors temptation b and different sensitivity r_a, from blue to red, the color bar indicates that the cooperation frequency changes from 0 to 1 accordingly. This figure aims to show the phase transition boundary of the system state more clearly than Fig. 3.

ratio. The different colors indicate the average activation ratio of the edges around the individual with the cooperation level corresponding to the y axis in the next iteration. In the SL network, where all individuals have only four edges, there were only five kinds of neighborhood cooperator proportions for individuals. First, it can be observed from the figure that in the early stage of evolution, when r_a is larger, individuals tend to exhibit more frequent social interaction in the subsequent round. This implies that in a harsh environment, more individuals may be forced to engage in social interaction. However, according to the parameter setting in Eq. (7), a larger value of r_a ought to render individuals more susceptible to their surroundings. Therefore, individuals with a larger proportion of defectors around them should exhibit less social interaction compared to those with a smaller value of r_a, but the actual situation is quite contrary, which is a counter-intuitive result. Furthermore, among the given parameter combinations, only when $r_a = 0.2$ did the evolution of individual interaction intensity exhibit a slow pace, while for $r_a = 0.6$ and $r_a = 3.0$, the interaction intensity stabilized after a significant number of iterations. Moreover, the overall tendency for social interaction on the network was observed to evolve

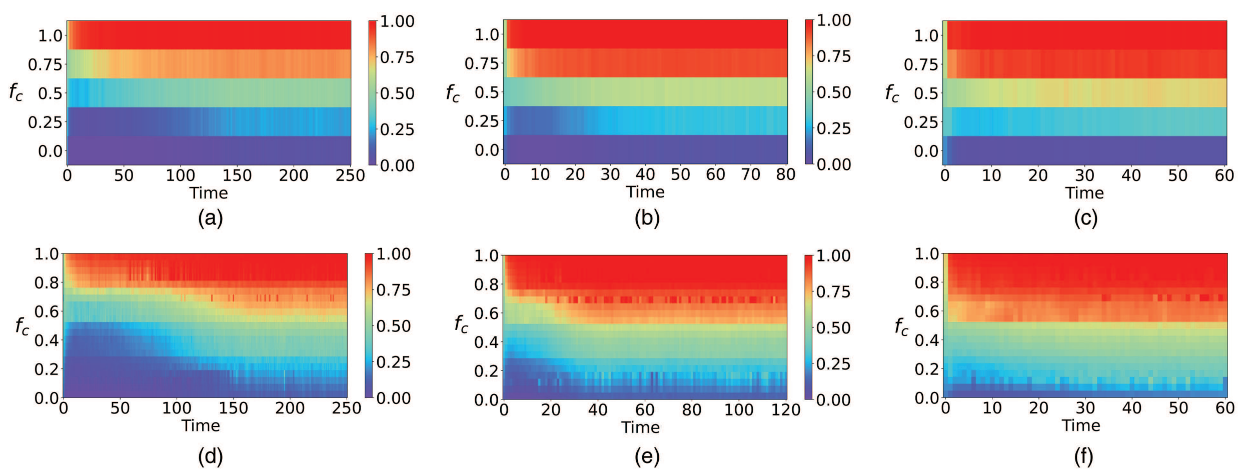


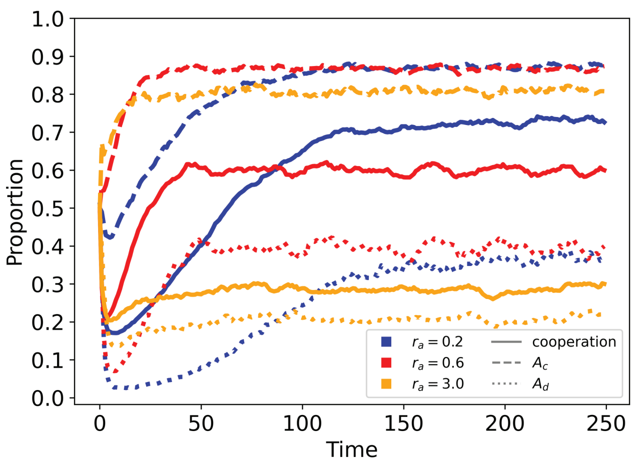
FIG. 5. The evolution of the activation ratio of edges connected to nodes with different neighborhood cooperation fractions over time for different parameter combinations (b, r_a). The activation ratio of edges of the next round of individuals with different fractions of cooperators f_c on their neighborhood is obtained on the SL network and the WS network under different parameter combinations. The y axis represents the different cooperator fractions f_c on one's neighborhood in one round, and the different colors from blue to red represent the proportion of activated edges on all the edges around the individual in the next iteration, and the x axis corresponds to the number of iterations. In all scenarios, the edge activation ratio of individuals undergoes an initial decline followed by a continuous increase. Specifically, when r_a is large, nodes in highly defective environments exhibit high edge activation rates, indicating that they are compelled to engage in social interactions in a harsh environment.

stronger in all circumstances, and even though the final fraction of cooperation varied for these parameter combinations, the distribution of social interaction intensity among individuals in different environments became relatively similar after the evolution reaching a stable state.

Moreover, we analyze the average social interaction ratios of both cooperators and defectors in each iteration for the three different r_a values, as illustrated in Fig. 6, and the change of the fraction of cooperation on the network is also put in the figure for comparison. We can find from the figure that in the early stage of evolution, the social interaction of the defectors decreases rapidly under all three parameter values. Meanwhile, for $r_a = 0.6$ and $r_a = 3.0$, even if the proportion of cooperation declines, the social interaction intensity of the cooperators increases rapidly until it approaches 1, in sharp contrast to the situation when $r_a = 0.2$. By comparing the social interaction patterns of cooperators and defectors, we can conclude that for $r_a = 0.6$ and $r_a = 3.0$, the number of cooperators on the network decreases rapidly, and their social interaction quickly approaches saturation. This indicates that the network rapidly forms a small, dense group of cooperators at the early stage of evolution, while the defectors remain in the sparse regions of the network. Due to the sensitivity of individuals' social interaction willingness, when r_a is higher, the cooperator group is denser than that when r_a is 0.2, which forms better protection for the cooperators. However, this also enables the defectors at the edge of the cooperator cluster to acquire more social interaction willingness, forcing the individuals around them to interact in a low-cooperation environment. This counter-intuitive result is clearly observed in Fig. 5, as previously mentioned. Furthermore, this interaction with defectors in a low-cooperation environment hinders the spread of cooperation, explaining why the cooperation strategy is difficult to spread even though the social interaction of cooperators is active when $r_a = 0.6$ and $r_a = 3.0$.

In addition, we have observed from Fig. 6 that for $r_a = 0.2$, although the social interaction ratio of cooperators is lower than that of the other two sets of r_a values in the early stage of evolution, the social interaction ratio of defectors is also significantly lower due to the lack of incentives from cooperators. This provides an opportunity for the cooperator cluster to gradually expand and spread, leading to an eventual increase in the fraction of cooperation. In Fig. 7, we present the average social interaction ratio of cooperators and defectors for more parameter combinations. We use the same set b values as in Fig. 6 and then show the average social activity intensity ratios of cooperators and defectors obtained by varying r_a. Our results suggest that when we fix the value of b, although f_c will change slightly due to the influence of r_a, the final social interaction ratios of cooperators and defectors only vary within a small range. These findings further support our explanation for why the fraction of cooperation when $r_a = 0.2$ is lower than that when r_a is high, especially when b is large. This is because the mutual assistance among cooperators is weaker in the sparse cooperator cluster formed by low social interaction willingness sensitivity, as compared with the dense cooperator cluster formed by high r_a, leading to lower reciprocity between collaborators. With a larger temptation to defect, the ability of the system to resist the invasion of defectors becomes weaker. Therefore, when b is large, forming a dense cooperator cluster rapidly, as with $r_a = 0.6$ and $r_a = 3.0$, is essential to effectively resist the invasion of defectors and maintain a high fraction of cooperation.

Considering that network size may have a significant impact on the evolution of cooperation, we conducted simulations at different network sizes to observe the robustness of our results to changes in



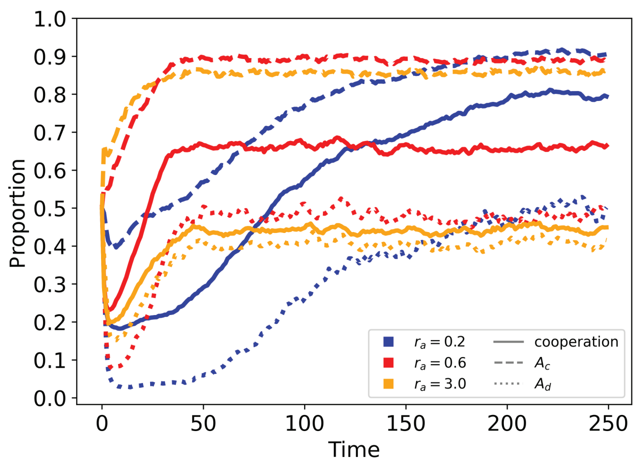


FIG. 6. Time evolution of the activation ratio of edges connected to cooperators and defectors for different r_a. The average social activity intensity of cooperators and defectors in the network changes with the number of iterations, obtained on the SL network and WS network under different parameter combinations. The intensity of social activity is calculated by the proportion of activated edges in the total number of edges around the individual. Blue, red, and yellow represent r_a of 0.2, 0.6, and 3.0, respectively, and dotted lines and dots represent the average intensity of social interaction of cooperators and defectors, respectively. In addition, we also put the curve of the fractions of cooperators in the figure for comparison, which is represented by solid lines. Initially, A_d rapidly decreases, when r_a is small, the activation ratio and cooperation fraction change slowly, which leads to a higher final cooperation proportion.

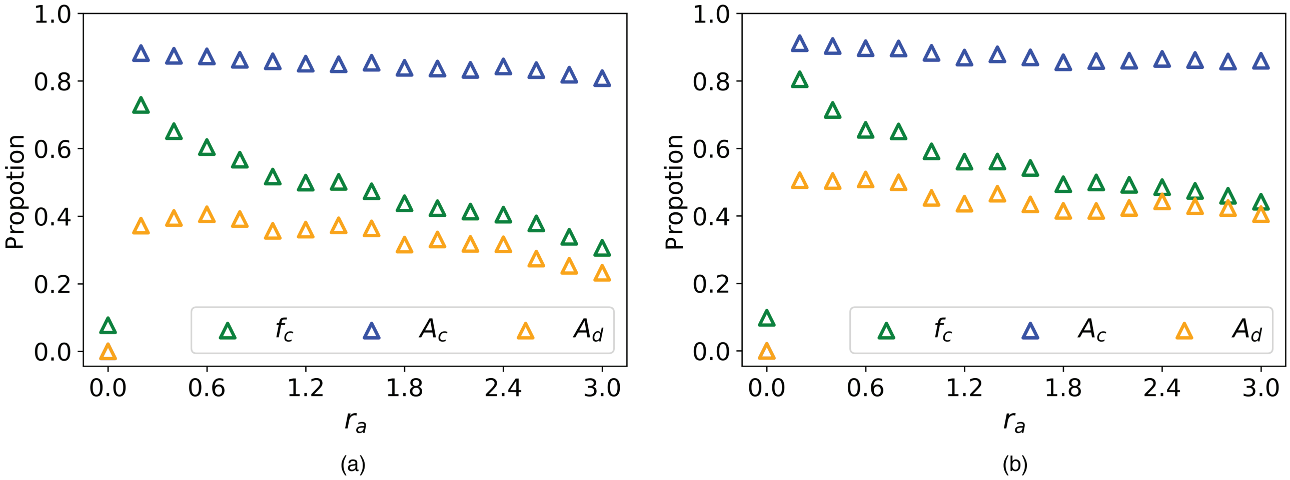


FIG. 7. The activity edge proportion of cooperators and defectors after evolution against r_a. The final average social activity intensity of cooperators and defectors after the evolution reaches stability, obtained on the SL network and the WS network in different parameter combinations. The defectors' temptation b is fixed to 0.45 on the SL network and 0.5 on the WS network. The yellow and blue dots represent the final social interaction intensity of the defector and cooperator, respectively, and the green dots represent the proportion of the cooperator obtained by the same parameter for comparison. As r_a increases from 0.2 to 3.0, f_c decreases continuously, while A_c and A_d remain almost unchanged when the evolution reaches stability.

network size. When the network size changes, the random reconnection probability and average degree of the WS network do not change. Figure 8 shows the trend of the change in the cooperation level f_c after the evolution reaches stability with respect to network size. Each result in the figure is captured after 500 steps of evolution. From the figure, we can observe that there is an impact for f_c when the network size is small. Specifically, when the network size is small and $r_a = 0.2$, f_c in the SL networks is much lower than that in other network sizes, and we can observe that the impact of network sizes on f_c in the SL networks is greater than that in the WS networks. Moreover, the cooperation level in the WS network is almost unaffected by changes in network size for any value of r_a and any network size, while the results in the SL network become almost unaffected when the number of nodes in the network exceeds 1600. This suggests that our results are unstable for smaller network sizes, but when the number of nodes is greater than 1600 in the SL

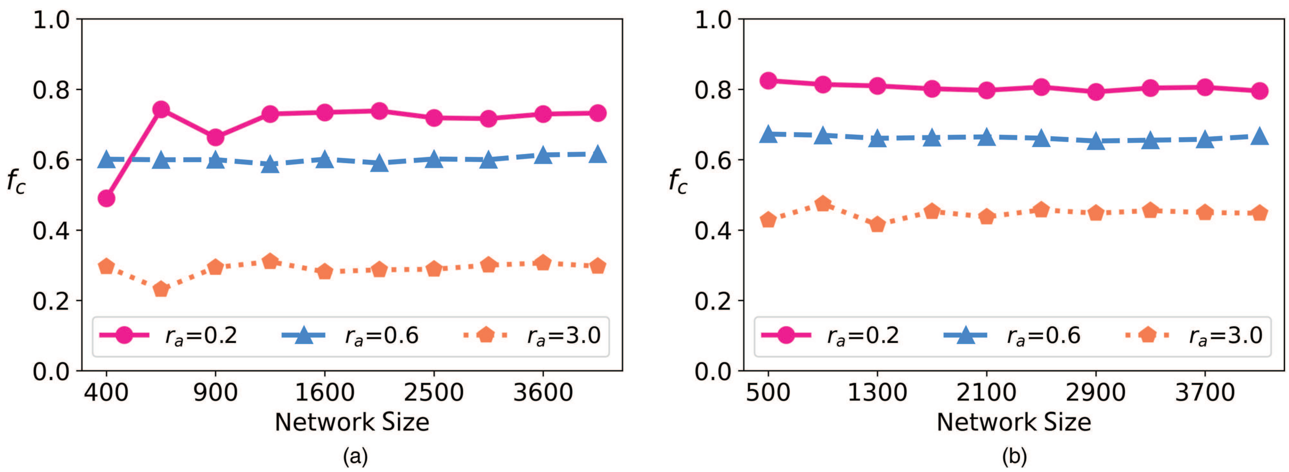


FIG. 8. The effect of network size on f_c after the system is stable. The influence of system scale on the final cooperation level f_c of WS networks and SL networks, where red, blue, and yellow represent the situation where r_a is 0.2, 0.6, and 3.0, respectively, and the temptation to defect b is set to 1.45 for SL networks and 1.50 for WS networks. The network scale ranges from 500 to 4100 on WS networks and from 400 to 4225 on SL networks. When the network size is small, the cooperation is greatly affected by the network size, but when the network size exceeds 1000, the evolutionary results become robust.

networks and greater than 1300 in the WS networks, while the network structure and type are unchanged, the results will be robust to network size.

IV. CONCLUSIONS AND OUTLOOK

In this study, we introduce a novel model, the social interaction willingness model, which incorporates the influence of an individual's game situation on the activation and inhibition of edges in subsequent iterations. We propose the social interaction willingness income matrix, using one parameter to measure the sensitivity of the individual's social interaction willingness to the environment. We apply this model to the weak prisoner's dilemma game on both the SL and WS networks and observe the evolution of different individuals' cooperation and social interaction intensity during the simulation. Our simulations reveal that when the temptation to defect is fixed, as long as the individual's social interaction is affected by the environment, at almost any level of social interaction willingness sensitivity, the fraction of cooperation at the equilibrium state of evolution is higher on dynamic networks than that on the static network under the same condition. When the temptation of the defector b is small, a lower sensitivity can lead to a higher final fraction of cooperation. Conversely, with an increase in b, the optimal sensitivity of social interaction willingness required to elicit the best fraction of cooperation also increases. The sensitivity of an individual's social interaction intention to the environment plays a critical role in determining the density and size of the cooperator cluster at the initial stage of network evolution. When the sensitivity parameter r_a is small, cooperation has less of a promotion effect on social interaction, resulting in sparse connections between cooperators on the network, making isolation of defectors more effective. As a result, the defectors' payoffs decrease, and cooperators can gradually contact other individuals and spread their strategy. However, because of the sparsity of cooperation clusters, when b is large, cooperators can hardly resist the invasion of defectors with higher payoffs. Therefore, low sensitivity performs well in environments with low defect temptation, but the fraction of cooperation is low when the temptation is larger. Conversely, when individuals' social interaction is more sensitive, the network quickly forms a small and dense cooperator cluster at the early stage of evolution, which is difficult for defectors to invade. However, the high level of sensitivity enables defectors at the edge of the cooperator cluster to gain significant social interaction willingness, reducing their isolation and giving them the ability to compel individuals around them to interact in a high-defect environment. This, coupled with the high temptation to defect, allows them to obtain high profits while interacting with individuals on the edge of the cooperator cluster, forming a blockade on the expansion of cooperation. This equilibrium makes it difficult to spread both cooperation and defect but protects the cooperators, keeping them safe from defectors and leading to a higher level of cooperation when the temptation is higher.

This study reveals the impact of local dynamics and heterogeneity of individual social interactions on cooperation within a given network. It is similar to previous research that utilizes evolving weights to model the unequal probability of individuals learning strategies from different neighbors.[44] However, our model differs from these studies in that individual fitness is not influenced by neighborhood profits and, during the coevolution of the network, individuals treat their neighbors equally, without actively manipulating interaction intensity with specific neighbors. Moreover, depending on the social interaction willingness of neighbors, one individual on the network can be forced to engage in the games. Furthermore, we focus solely on the weak prisoner's dilemma model and Fermi updating rules, although other models, such as the snowdrift game and deer hunting game, could also be applied, as well as alternative strategy updating rules such as the birth and death process or learning optimal strategies. Varying these factors could yield diverse results, highlighting the complexity of cooperative behavior in evolving networks. Last, it is worth mentioning that recent studies have pointed out that cooperative behavior is just one facet of morality that could be studied using evolutionary game models,[46] we hope our research will contribute to future research on evolutionary games on networks and moral preferences.

ACKNOWLEDGMENTS

This work was supported in part by the National Nature Science Foundation of China (NNSFC) under Grant No. 62206230 and in part by the Humanities and Social Science Fund of Ministry of Education of the People's Republic of China under Grant No. 21YJCZH028.

AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Yichao Yao: Conceptualization (equal); Methodology (equal); Writing – original draft (equal). **Ziyan Zeng:** Validation (equal); Writing – review & editing (equal). **Bin Pi:** Validation (equal); Writing – review & editing (equal). **Minyu Feng:** Supervision (equal); Validation (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

[1] J. Hofbauer, K. Sigmund *et al.*, *Evolutionary Games and Population Dynamics* (Cambridge University Press, 1998).
[2] M. Nowak and R. Highfield, *Supercooperators: Altruism, Evolution, and Why We Need Each Other to Succeed* (Simon and Schuster, 2011).
[3] M. Perc, J. J. Jordan, D. G. Rand, Z. Wang, S. Boccaletti, and A. Szolnoki, "Statistical physics of human cooperation," Phys. Rep. **687**, 1–51 (2017).
[4] J. M. Smith and G. R. Price, "The logic of animal conflict," Nature **246**(5427), 15–18 (1973).
[5] G. Szabó and G. Fáth, "Evolutionary games on graphs," Phys. Rep. **446**(4-6), 97–216 (2007).
[6] J. Tanimoto, *Evolutionary Games with Sociophysics*, Evolutionary Economics Vol. 17 (Springer, 2019).
[7] M. A. Nowak, "Five rules for the evolution of cooperation," Science **314**(5805), 1560–1563 (2006).

[8] E. Lieberman, C. Hauert, and M. A. Nowak, "Evolutionary dynamics on graphs," Nature **433**(7023), 312–316 (2005).
[9] M. Doebeli and C. Hauert, "Models of cooperation based on the prisoner's dilemma and the snowdrift game," Ecol. Lett. **8**(7), 748–766 (2005).
[10] M. Perc, J. Gómez-Gardenes, A. Szolnoki, L. M. Floría, and Y. Moreno, "Evolutionary dynamics of group interactions on structured populations: A review," J. R. Soc. Interface **10**(80), 20120997 (2013).
[11] J. Poncela, J. Gómez-Gardenes, L. M. Floría, and Y. Moreno, "Robustness of cooperation in the evolutionary prisoner's dilemma on complex networks," New J. Phys. **9**(6), 184 (2007).
[12] C. Hauert and M. Doebeli, "Spatial structure often inhibits the evolution of cooperation in the snowdrift game," Nature **428**(6983), 643–646 (2004).
[13] Q. Su, A. Li, L. Zhou, and L. Wang, "Interactive diversity promotes the evolution of cooperation in structured populations," New J. Phys. **18**(10), 103007 (2016).
[14] D. Jia, X. Wang, Z. Song, I. Romić, X. Li, M. Jusup, and Z. Wang, "Evolutionary dynamics drives role specialization in a community of players," J. R. Soc. Interface **17**(168), 20200174 (2020).
[15] Z. Wang, A. Szolnoki, and M. Perc, "Self-organization towards optimally interdependent networks by means of coevolution," New J. Phys. **16**(3), 033041 (2014).
[16] M. A. Nowak and R. M. May, "Evolutionary games and spatial chaos," Nature **359**(6398), 826–829 (1992).
[17] M. A. Nowak and R. M. May, "The spatial dilemmas of evolution," Int. J. Bifurcation Chaos **3**(01), 35–78 (1993).
[18] G. Szabó and C. Tőke, "Evolutionary prisoner's dilemma game on a square lattice," Phys. Rev. E **58**(1), 69 (1998).
[19] F. C. Santos and J. M. Pacheco, "Scale-free networks provide a unifying framework for the emergence of cooperation," Phys. Rev. Lett. **95**(9), 098104 (2005).
[20] J. Poncela, J. Gómez-Gardeñes, L. M. Floría, Y. Moreno, and A. Sánchez, "Cooperative scale-free networks despite the presence of defector hubs," Europhys. Lett. **88**(3), 38003 (2009).
[21] J. Gómez-Gardeñes, J. Poncela, L. Mario Floría, and Y. Moreno, "Natural selection of cooperation and degree hierarchy in heterogeneous populations," J. Theor. Biol. **253**(2), 296–301 (2008).
[22] N. Masuda and K. Aihara, "Spatial prisoner's dilemma optimally played in small-world networks," Phys. Lett. A **313**(1-2), 55–61 (2003).
[23] B. J. Kim, A. Trusina, P. Holme, P. Minnhagen, J. S. Chung, and M. Y. Choi, "Dynamic instabilities induced by asymmetric influence: Prisoners' dilemma game in small-world networks," Phys. Rev. E **66**(2), 021907 (2002).
[24] M. Jusup, P. Holme, K. Kanazawa, M. Takayasu, I. Romić, Z. Wang, S. Geček, T. Lipić, B. Podobnik, L. Wang, W. Luo, T. Klanjšček, J. Fan, S. Boccaletti, and M. Perc, "Social physics," Phys. Rep. **948**, 1–148 (2022).
[25] A. Szolnoki, M. Perc, and Z. Danku, "Towards effective payoffs in the prisoner's dilemma game on scale-free networks," Physica A **387**(8-9), 2075–2082 (2008).
[26] J. M. Pacheco, A. Traulsen, and M. A. Nowak, "Coevolution of strategy and structure in complex networks with dynamical linking," Phys. Rev. Lett. **97**(25), 258103 (2006).
[27] M. Perc and A. Szolnoki, "Coevolutionary games—A mini review," BioSystems **99**(2), 109–125 (2010).
[28] M. G. Zimmermann, V. M. Eguiluz, and M. S. Miguel, "Cooperation, adaptation and the emergence of leadership," in *Economics with Heterogeneous Interacting Agents* (Springer, 2001), pp. 73–86.
[29] H. Ebel and S. Bornholdt, "Evolutionary games and the emergence of complex networks," cond-mat/0211666 (2002).
[30] J. M. Pacheco, A. Traulsen, and M. A. Nowak, "Active linking in evolutionary games," J. Theor. Biol. **243**(3), 437–443 (2006).
[31] D. G. Rand, S. Arbesman, and N. A. Christakis, "Dynamic social networks promote cooperation in experiments with humans," Proc. Natl. Acad. Sci. **108**(48), 19193–19198 (2011).
[32] A. Szolnoki, M. Perc, and Z. Danku, "Making new connections towards cooperation in the prisoner's dilemma game," Europhys. Lett. **84**(5), 50007 (2008).
[33] A. Szolnoki and M. Perc, "Resolving social dilemmas on evolving random networks," Europhys. Lett. **86**(3), 30007 (2009).
[34] A. Szolnoki and M. Perc, "Emergence of multilevel selection in the prisoner's dilemma game on coevolving random networks," New J. Phys. **11**(9), 093033 (2009).
[35] C. Gracia-Lázaro, L. M. Floría, J. Gómez-Gardeñes, and Y. Moreno, "Cooperation in changing environments: Irreversibility in the transition to cooperation in complex networks," Chaos, Solitons Fractals **56**, 188–193 (2013).
[36] A. Szolnoki and X. Chen, "Environmental feedback drives cooperation in spatial social dilemmas," Europhys. Lett. **120**(5), 58001 (2018).
[37] Z. Song, H. Guo, D. Jia, M. Perc, X. Li, and Z. Wang, "Reinforcement learning facilitates an optimal interaction intensity for cooperation," Neurocomputing **513**, 104–113 (2022).
[38] W. Li, X. Zhang, and G. Hu, "How scale-free networks and large-scale collective cooperation emerge in complex homogeneous social systems," Phys. Rev. E **76**(4), 045102 (2007).
[39] H. Sayama, I. Pestov, J. Schmidt, B. J. Bush, C. Wong, J. Yamanoi, and T. Gross, "Modeling complex systems with adaptive networks," Comput. Math. Appl. **65**(10), 1645–1664 (2013).
[40] Z. Wang, M. Jusup, L. Shi, J.-H. Lee, Y. Iwasa, and S. Boccaletti, "Exploiting a cognitive bias promotes cooperation in social dilemma experiments," Nat. Commun. **9**(1), 2954 (2018).
[41] Z. Wang, M. Jusup, H. Guo, L. Shi, S. Geček, M. Anand, M. Perc, C. T. Bauch, J. Kurths, S. Boccaletti, and H. J. Schellnhuber, "Communicating sentiment and outlook reverses inaction against collective risks," Proc. Natl. Acad. Sci. U.S.A. **117**(30), 17650–17655 (2020).
[42] J. Sander, J. Schupp, and D. Richter, "Getting together: Social contact frequency across the life span," Dev. Psychol. **53**(8), 1571 (2017).
[43] K. T. D. Eames, N. L. Tilston, E. Brooks-Pollock, and W. J. Edmunds, "Measured dynamic social contact patterns explain the spread of H1N1v influenza," PLoS Comput. Biol. **8**(3), e1002425 (2012).
[44] C. Shen, C. Chu, H. Guo, L. Shi, and J. Duan, "Coevolution of vertex weights resolves social dilemma in spatial networks," Sci. Rep. **7**(1), 15213 (2017).
[45] G. Szabó, J. Vukov, and A. Szolnoki, "Phase diagrams for an evolutionary prisoner's dilemma game on two-dimensional lattices," Phys. Rev. E **72**(4), 047107 (2005).
[46] V. Capraro and M. Perc, "Mathematical foundations of moral preferences," J. R. Soc. Interface **18**(175), 20200880 (2021).