Evolution of Cooperation in the Memory-based Multigame on Complex Networks

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***Abstract*—Understanding the emergence of cooperation in society is an open problem. In response to this problem, the network evolutionary game and its derivatives, such as multigame, have been shown to be an effective method to address social dilemmas. In this paper, we investigate the memory-based multigame consisting of the prisoner's dilemma game and the snowdrift game on the square lattice with periodic boundary and small-world networks. Plus, we also propose a novel strategy-updating rule by combining the player's memory mechanism and replicator dynamics. Subsequently, we conduct simulations to demonstrate the effects of the payoff parameters and players' memories on network cooperation behavior. Our results show that by introducing the memory mechanisms and multigame, cooperation can occur widely on complex networks.**

Keywords—Evolutionary game, Memory-based multigame, Complex networks, Coevolution

I. INTRODUCTION

Understanding how cooperative behavior evolves has been an ongoing effort that has attracted the attention of researchers from a variety of disciplines [1]. The emergence of evolutionary game theory offers a powerful mathematical framework for exploring the underlying mechanisms for solving cooperative dilemmas [2], which seeks to mathematically capture behavior in strategic situations in which the success of a player in making choices depends on the preferences of others [3]. In addition, various mechanisms have been proposed to explain the emergence of cooperation in selfish groups, the most famous of which are the five rules proposed by Nowak in 2006 [4], including indirect reciprocity [5], direct reciprocity [6], kin selection [7], network reciprocity [8], and group selection [9]. Meanwhile, the memory mechanisms of individuals also play a crucial role in the gaming process, i.e., in reality, individuals update their strategies usually not only considering their own payoffs, but also the strategies they have adopted in the past [10].

Complex networks, as quantification of real complex systems, have developed rapidly in the past few decades. With the introduction of the concept of scale-free networks [11], researchers begin to realize the significance of network science and gradually achieve fruitful results. A variety of network models and their topological properties [12][13][14] were springing up after the scale-free network was put forward. For example, Boccaletti et al. [15] provided a comprehensive review of the structural and dynamical organization of graphs that are composed of different relationships (layers) between their constituents. Feng et al. [16] proposed a subnormal distribution deduced from the evolutionary networks with variable elements and investigated its statistical properties to better characterize the real distributions. Moreover, the temporal networks [17][18] and higher-order networks [19][20] have also developed rapidly in recent years. Furthermore, the existing research on the evolutionary game has also yielded abundant results on these novel network models, which can be summarized as cooperative behavior can appear on different network structures. For instance, Pi et al. [21] investigated the evolutionary multigame with conformists and profiteers on dynamic complex networks. Zeng et al. [22] introduced a network model regarding the $M/M/\infty$ queuing system and studied the weak prisoner's dilemmas.

Recently, some researchers have noticed that there should not be one game in the population, i.e., the players in the network are playing the multigame [23][24]. Additionally, some studies have been conducted on players' memory mechanisms and found that the emergence of cooperative behavior in the network can be significantly facilitated by concerning memory mechanisms. For example, Shu et al. [25] proposed a new memory-based method and discovered that high cooperation levels simultaneously emerged for both small and large cost-to-benefit ratios. Based on the aforementioned points, in this paper, we focus on the player's memory mechanism, which ensures that the players remember the strategies they have utilized in the past and take them into account when updating their strategies. Besides, the game played by the players in the network is not just a single game, but a multigame consisting of the prisoner's dilemma game and snowdrift game.

The structure of the paper is as follows: We describe our multigame model and illustrate the strategy-updating rule of players regarding the memory mechanism in Section II. In Section III, we carry out numerical simulations based on the constructed model and display the simulation results. Lastly, we conclude this paper and depict the outlook in Section IV.

II. MODEL

In this section, we mainly present multiple games in the network, with each player obtaining different payoffs based on the game model they play. Subsequently, players will update their strategies regarding their past memories and their pursuit of maximum payoffs.

A. The Multigame Model and Calculation of Players' Payoffs

We first briefly introduce the prisoner's dilemma game (PDG) and the snowdrift game (SDG) we will use in the model. In PDG, consider two thieves who work in partnership and are arrested and interrogated in isolation. If both confess to the crime (defection), then both gain the payoff of 0, called the "punishment for mutual defection", while if both refuse to confess (cooperation), they both gain the payoff of 1, called the "reward for mutual cooperation". However, if one confesses (defection) and the other refuses to confess (cooperation), the former gain the payoff of b, called the "temptation to defect", whereas the latter gain the payoff of $-\delta$, called the "suckers's payoff". Thus, we can denote the payoff matrix of the PDG as below:

$$M1 = \begin{pmatrix} 1 & -\delta \\ b & 0 \end{pmatrix}. \tag{1}$$

In SDG, consider two drivers trapped on either side of a large avalanche in a snowstorm, they now have two choices: either get out of the car and shovel the snow (cooperation) or stay in the car and do nothing (defection). If both drivers are willing to get out of the car and shovel the snow, both get the payoff of 1 per person since they are able to get home and share the cost of shoveling the snow. If both drivers stay in the warm car, they do not get home on time, i.e., both get the payoff of 0. However, if one of them gets out of the car and shovels, they both get home, while the shoveling driver (cooperator) will have the job of shoveling alone and thus gets the payoff of δ. The driver staying in the car and doing nothing (defector) gets home without having to work and thus gets the maximum payoff of b. Therefore, according to the practical meaning of the SDG description, we can express the payoff matrix of the SDG as below:

$$M2 = \begin{pmatrix} 1 & \delta \\ b & 0 \end{pmatrix}, \tag{2}$$

where $1 \le b \le 2$ and $0 \le \delta \le 1$ are two adjustable parameters in Eqs. 1 and 2. Generally, defectors will dominate the network as $b \to 2$, and cooperators will dominate the network as $b \to 1$.

We investigate the multigame on a network with a group of N players consisting of defectors and cooperators, where each player performs a particular game with all neighbors, either the PDG or the SDG. Besides, we introduce a parameter $\rho \in [0,1]$ to indicate the fraction of players that play SDG, with the remaining players playing PDG. Naturally, it will change to the traditional PDG when $\rho = 0$, and to the traditional SDG when $\rho = 1$.

B. The Strategy Evolution of Player

In this subsection, we illustrate the rule for updating strategies that combines players' memory and the pursuit of maximum benefit. At each time step, the players in the network update their strategies synchronously, i.e., each player will decide the strategy to adopt at the next step based on his/her own memory and payoff. In reality, rational payers usually take the strategies they used in the past into account when updating their strategies, which means that they all have memories. Besides, a rational player has a tendency to chase maximum payoff, i.e. he/she prefers to adopt a strategy with high payoff among his/her neighbors, which is portrayed by replicator dynamics. Concretely, a game player obtains payoff by playing with all neighbors. And when a game player x decides to update his/her strategy, he/she will randomly choose one of his/her neighbors y to compare payoff, and if the payoff U_y of the neighbor y is greater than the payoff U_x of the player x, i.e., $U_y > U_x$, then the player x with memory mechanism will imitate the strategy of player y in the next game with a probability considering the strategies utilized in the past and the payoff difference between player x and player y is expressed as follows:

$$P = c\frac{\sum_{i=1}^{T} \delta\left(S_{x,T}(i), s_y\right)}{T} + (1-c)\left[\frac{U_y - U_x}{D \cdot \max(k_x, k_y)}\right]_0^1. \tag{3}$$

Herein, s_i indicates the strategy of the player i, c represents the memory strength, $S_{x,T}$ denotes the strategy set of the player x in the past T game steps, and we can define $\delta(S_{x,T}(i), s_y)$ as below:

$$\delta(S_{x,T}(i), s_y) = \begin{cases} 1, & S_{x,T}(i) = s_y \\ 0, & S_{x,T}(i) \neq s_y \end{cases}, \tag{4}$$

which is a Dirac function. It is worth noting that we consider the memory length of the player to be t as $t < T$ since the evolution time has not yet reached T currently. Moreover, $max(k_x, k_y)$ represents the greater degree of player x and player y, and D denotes the difference between the maximum parameter and the minimum parameter in the payoff matrix, which means $D = b + \delta$ for PDG while $D = b$ for SDG. Besides, the operator $[z]_0^1$ is defined as follows:

$$[z]_0^1 = \begin{cases} 0, z \leqslant 0 \\ z, 0 < z < 1. \\ 1, z \geqslant 1 \end{cases} \tag{5}$$

The effect of Eq. 5 is to keep the probability P that a player imitates a neighbor's strategy in the range of 0 to 1 since the equation $\frac{U_y - U_x}{D \cdot \max(k_x, k_y)}$ may be greater than 1 or less than 0.

III. Simulations and Discussions

In this section, we present our simulation methods and results to demonstrate the effect of some parameters in the model on the fraction of network cooperation (f_c), which is denoted as the number of cooperators in the network divided by the size of the network.

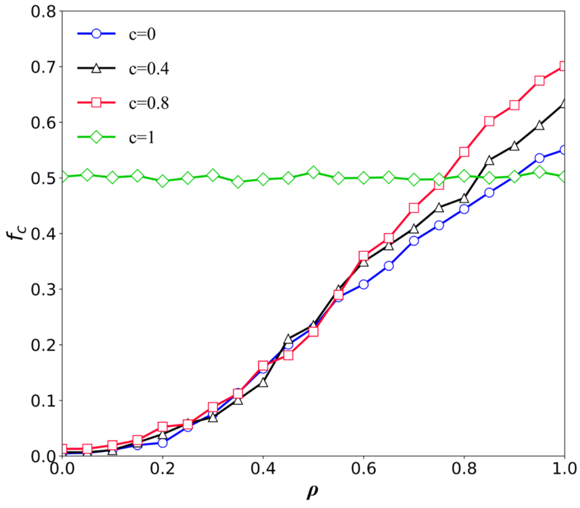
A. Methods

The simulation is implemented in Python. We carry out the simulation on 2 networks, including the square lattice with periodic boundary (SL) and the WS network, which is proposed by Watts and Strogatz in 1998 [26]. In this paper, we utilize the function $watts\_strogate\_graph()$ of $networkx$ in Python to generate the WS network. At the beginning of the simulation, the WS network has 1000 nodes and the SL network has 32×32 nodes, where each node represents a player, and the connected edges between nodes indicate the events related to interactions. Moreover, the player's strategy at the beginning is randomly assigned to cooperate or defect, and the percentage of SDG players in the network is ρ. Additionally, we perform the evolution steps of cooperation with a length of $T = 10^4$ steps to ensure that the fraction of cooperation in the network reaches a smooth state. In order to avoid additional disturbances, we average 5 independent simulations for each set of parameters to obtain the final results.

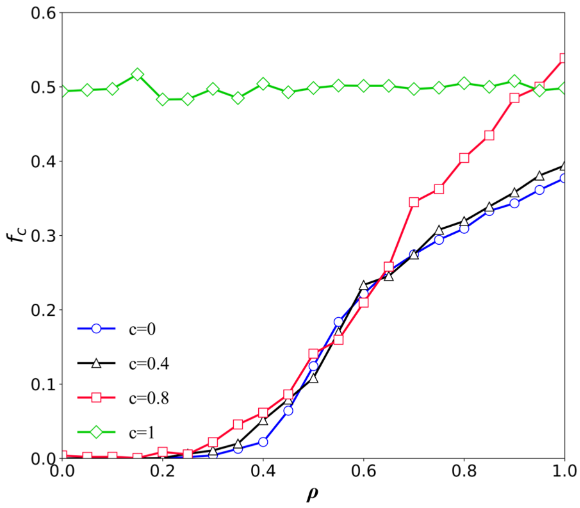
B. The Influence of the Memory Strength and Proportion of SDG Performed on Network Cooperation Behavior

In this simulation, we investigate the impact of the proportion ρ of SDG performed on network cooperation behavior. By fixing the memory length $T = 5$, the payoff parameters $b = 1.3$, and $\delta = 0.3$, we show the function of cooperation frequency f_c and proportion of SDG performed $\rho \in [0, 1]$ under the conditions of memory strength c = 0, 0.4, 0.8, and 1 respectively in Fig. 1. Both WS and SL networks exhibit an increase in the frequency of cooperation as the proportion of SDG performed in the network grows, except $c = 1$, that will be discussed later. Besides, as shown in both Figs. 1(a) and 1(b), both networks illustrate a greater memory strength c causing a higher proportion of cooperation f_c as ρ is large enough ($\rho > 0.7$), although this phenomenon is not significant at $\rho < 0.7$. The difference between Figs. 1(a) and 1(b) is that under the same conditions, the WS network will be more conducive to the emergence of cooperation than the SL network since the frequency of cooperation f_c is greater in the WS network (Fig. 1(a)) than that in the SL network (Fig. 1(b)) as the memory strength c and proportion of SDG performed ρ are the same.

Herein, we explain the case of $c = 1$ separately in Fig. 1. As indicated in Eq. 3, it will become $P(s_x \leftarrow s_y) = \frac{\sum_{i=1}^{T} \delta(S_{x,T}(i), s_y)}{T}$ as $c = 1$, which means that the player's strategy-updating rule at this point is only related to the initial strategy of the game, but not to the strategies of the neighbors and their payoffs, that is to say, the player's strategy will not change as time progresses. Therefore, the density of cooperation will fluctuate around 0.5 with the change of ρ when $c = 1$, since the initial setting of each round of the game is that the probability of each player choosing to cooperate or defect equals 0.5.



(a) WS

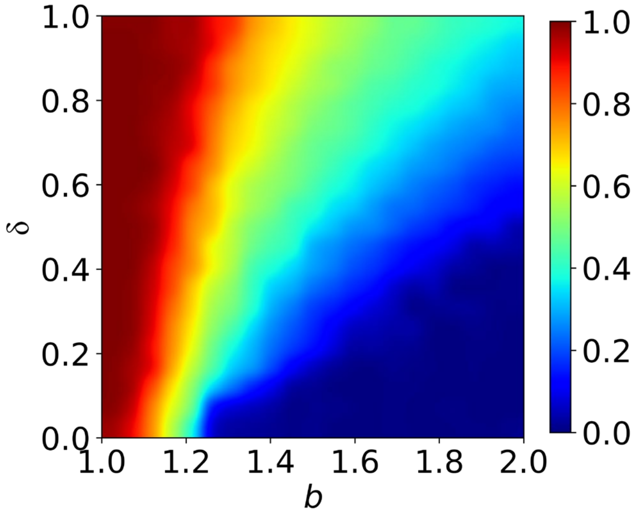


(b) SL

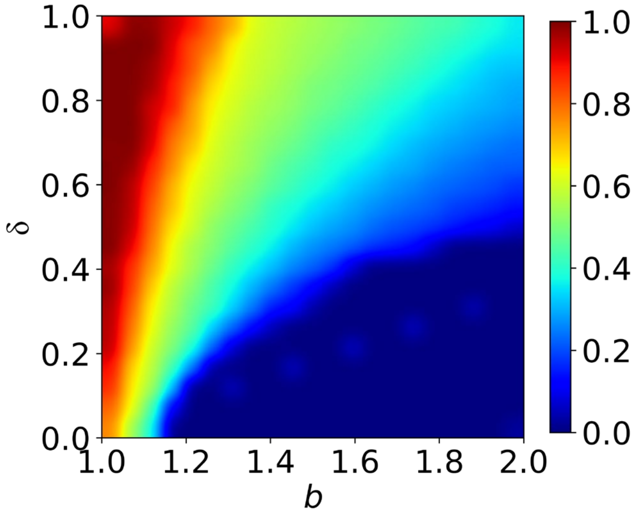
Fig. 1. The influence of the memory strength and proportion of SDG performed on network cooperation behavior. We set the ratio ρ conducting SDG to [0, 1] and observe the evolution of cooperation in WS (in panel (a)) and SL (in panel (b)) networks under the conditions of memory strength c = 0, 0.4, 0.8, and 1, respectively. Both networks, except for c = 1, demonstrate that the frequency of cooperation in the network grows as the fraction ρ of SDG performed increases.

C. The Influence of the Payoff Parameters on Network Cooperation Behavior

Subsequently, we set the x-axis as $b \in [1, 2]$ and the y-axis as $\delta \in [0, 1]$ to further explore the effect of the payoff parameters on the frequency of network cooperation. The heat map of the cooperation frequency with payoff parameters on the WS network is demonstrated in Fig. 2(a), from which we can derive that a small b or a large δ can promote the cooperation density in the WS network. Next, we display the relationship between payoff parameters and cooperation frequency on the SL network in Fig. 2(b), where the memory length $T=5$, memory strength $c=0.2$, and proportion of SDG performed $\rho=0.8$. We can yield the results that the frequency of cooperation increases with δ while decreasing with b, which is similar to the WS network shown in Fig. 2(a). Furthermore, the WS network enhances cooperation better than the SL network by comparing Figs. 2(a) and 2(b), which is consistent with our previous analysis. We note that although a few color regions in the heat maps of Fig. 2 are not ordered, and the possible reason is the randomness of player decisions, the effect of the previously discussed payoff parameters on network cooperation is not significantly affected.



(a) WS



(b) SL

Fig. 2. The influence of the payoff parameters on network cooperation behavior. We set the payoff parameters b and δ to [1, 2] and [0, 1], respectively to study the evolution of cooperation in WS (in panel (a)) and SL (in panel (b)) networks. Both networks display that the cooperation will be facilitated as δ grows, whereas will be inhibited as b increases.

Then, we perform an analysis of the causes of the previously mentioned phenomena. As demonstrated in Eqs. 1 and 2, the payoff of the defector will increase when the payoff parameter b increases, which will result in more players in the network adopting the defection strategy. Therefore, we can derive that a greater b affords a smaller cooperation density. Moreover, for the PDG player, the payoff of the cooperation will decrease when the payoff parameter δ increases, however, for the SDG player, the payoff of the cooperator will be promoted as the payoff parameter δ grows. Plus, the proportion ρ of players performing SDG in the simulation of Fig. 2 is equal to 0.8. Consequently, we can deduce that the cooperative behavior in the network will be enhanced by increasing the payoff parameter δ.

IV. Conclusions and Outlooks

In this paper, we investigate the multigame (PDG and SDG) between players with memory mechanisms on the square lattice networks with periodic boundary and small-world networks. Initially, we introduce the traditional PDG and SDG and illustrate the calculation of the payoff based on the game played by the players. Subsequently, we propose a new rule of strategy updating regarding the player's memory mechanism and the nature of the quest for maximum payoff. In the simulation, we explore the impact of the memory strength and proportion of SDG performed on the emergence of network cooperation, and we find that cooperators can be facilitated by improving the memory strength of players or by increasing the proportion of SDG performed in the network. In addition, the influence of the payoff parameters in the game on the proportion of network cooperation is examined in the form of heat maps, and we discover that a larger value of b inhibits the emergence of cooperators in the network. Although the percentage of cooperation grows by improving δ, the possible reason for this is due to the high ratio of SDG performed in the network. However, very different results may occur when the proportion of SDG performed in the network is small, i.e., the proportion of cooperators will decrease as δ grows since the fact that a larger δ in PDG will afford a more negative payoff for cooperators.

In our work, the multigame we studied is only based on PDG and SDG, and there are some other games that can be investigated, such as the public goods game, stag hunt game, etc. Plus, there are also some extensions to study based on our research. For example, we explore the cooperation frequency in two network types by the replicator dynamics. Different strategy-updating rules, such as the Femi process, the imitate the best, and the birth-death process, may yield new results. Finally, we hope that our study will contribute to future research related to the network evolutionary game.

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