RESEARCH ARTICLE | FEBRUARY 07 2024

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Chaos 34, 023109 (2024)
https://doi.org/10.1063/5.0192619

Chaos ARTICLE pubs.aip.org/aip/cha

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Cite as: Chaos **34**, 023109 (2024); doi: 10.1063/5.0192619
Submitted: 20 December 2023 · Accepted: 11 January 2024 ·
Published Online: 7 February 2024

View Online Export Citation CrossMark

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ABSTRACT

The emergence of the evolutionary game on complex networks provides a fresh framework for studying cooperation behavior between complex populations. Numerous recent progress has been achieved in studying asymmetric games. However, there is still a substantial need to address how to flexibly express the individual asymmetric nature. In this paper, we employ mutual cognition among individuals to elucidate the asymmetry inherent in their interactions. Cognition arises from individuals' subjective assessments and significantly influences their decision-making processes. In social networks, mutual cognition among individuals is a persistent phenomenon and frequently displays heterogeneity as the influence of their interactions. This unequal cognitive dynamic will, in turn, influence the interactions, culminating in asymmetric outcomes. To better illustrate the inter-individual cognition in asymmetric snowdrift games, the concept of favor value is introduced here. On this basis, the evolution of cognition and its relationship with asymmetry degree are defined. In our simulation, we investigate how game cost and the intensity of individual cognitive changes impact the cooperation frequency. Furthermore, the temporal evolution of individual cognition and its variation under different parameters was also examined. The simulation results reveal that the emergence of heterogeneous cognition effectively addresses social dilemmas, with asymmetric interactions among individuals enhancing the propensity for cooperative choices. It is noteworthy that distinctions exist in the rules governing cooperation and cognitive evolution between regular networks and Watts–Strogatz small-world networks. In light of this, we deduce the relationship between cognition evolution and cooperative behavior in co-evolution and explore potential factors influencing cooperation within the system.

Evolutionary game theory on networks has gained significant attention for investigating cooperative behavior within groups. We propose utilizing mutual cognition among individuals in society to characterize the asymmetry in their interactions. The reciprocal influence between game strategies and mutual cognition facilitates co-evolution, ultimately leading to heterogeneous cognition and stable cooperative outcomes. Additionally, perspectives from network topological property and the degree of cognition evolution provide possible explanations for the simulation results based on two types of networks. Our work may offer new insights into forms of reflecting asymmetry in game interactions.

I. INTRODUCTION

Cooperation plays a significant role in human society and biology,[1,2] a concept seemingly incongruent with the theoretical framework put forth by Charles Darwin.[3,4] The question of why self-interested individuals are willing to bear the cost to benefit the group remains without a definitive solution. Explaining the cooperative behavior of self-interested individuals in a system continues to pose a challenge.[5] Evolutionary game theory, designed to solve biological problems, has aroused wide interest in economics,[6,7] physics,[8–10] and psychology,[11] providing a powerful and feasible framework for studying cooperation in this situation.[12] Nowak has explained human cooperative behavior from the perspective of game theory, forming five basic theories: kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection.[13,14] Among them, the prisoner's dilemma game (PDG)[15] and the snowdrift game (SDG)[16,17] are often mentioned as examples to describe pairwise interactions. In addition, complex networks provide a new way to study this problem. Evolutionary games on different network populations have been widely proposed, including small-world,[18] lattice,[19,20] and scale-free networks.[21]

Furthermore, the study of evolutionary games on multilayer networks,[22,23] higher-order networks,[24] and temporal networks[25] have been recent extensions.

It is widely acknowledged that the presence of spatial structure facilitates the emergence of cooperative clusters in PDG,[19] thereby countering the invasion of defectors. However, for SDG, it should be noted that the spatial structure sometimes inhibits the emergence of cooperation.[17] Within a group, individuals may pursue their self-interests; yet if everyone adopts an identical strategy, it can ultimately lead to resource depletion or even extinction for the entire group. This phenomenon could be commonly observed in various social contexts such as resource overexploitation, environmental degradation, and misuse known as the "tragedy of the commons." To solve these issues, numerous mechanisms[26,27] that can promote cooperation, such as reward,[28,29] punishment,[30,31] multi-person group interactions,[32] different social roles,[33,34] and reputation,[35,36] have been extensively studied within the framework of spatial evolutionary game theory.

In conventional models of evolutionary game theory, all participants are considered homogeneous, irrespective of their status or cognitive relationship, and they employ the same game matrix.[12,37] However, it is widely acknowledged that no two individuals in the world are identical. There always exist variations among individuals, and even slight disparities can exert a significant impact on the outcome of their interactions. Numerous experimental studies have demonstrated that interactions between individuals often display asymmetry, which is also observed in natural settings among individuals or populations.[38,39] Moreover, extensive investigations have been conducted on asymmetric interactions influenced by individual characteristics or heterogeneous resource distribution.[40,41] However, the complexity of interactions between individuals requires further understanding. Over the past few years, there have been studies that focused on the effect of variable properties of game payoff on the evolution of cooperation.[42–44] To capture the ever-changing environment, Su *et al.* considered a model of evolutionary dynamics with game transitions,[45] which reveals that simple game transitions can promote prosocial behaviors. Zeng *et al.* proposed a stochastic payoff game matrix that makes each individual's payoff follows a specific probability distribution with a fixed expectation,[46] finding the impact of the changing payoff on cooperation.

However, a disparity exists between the content above and our grasp of reality, with the heterogeneities of these individuals traditionally regarded as immutable attributes in an idealized context. It is widely recognized that individual traits are dynamic, and factors like height, weight, position, and education, among others, are subject to change over time.[47] Therefore, it becomes imperative to aptly depict dynamic properties that encapsulate the diverse characteristics of individuals. In such instances, the flexible alteration of heterogeneous features in individuals gives rise to corresponding variations in asymmetric interactions.[48] Actually, even colleagues in identical positions can exhibit asymmetric cognition and interaction. Whatever the status, distinct individuals nurture varied perceptions of the same person, encapsulating the essence of "Ten people, ten minds." In consideration of the preceding discussions, this paper introduces an asymmetric snowdrift game originating from the variable heterogeneous cognition among individuals. We forego the direct delineation of individual characteristics and, instead, reflect the characteristics of individuals through inter-individual cognition, where intricate interactions within a system can engender evolution in cognition among individuals. This interaction can manifest as alterations in individual traits or as other forms of moral human behavior distinct from cooperation.[49] For instance, individuals may extend help to others driven by self-interest, be it material rewards or spiritual satisfaction. Furthermore, when individuals receive assistance, a sense of gratitude often emerges toward their helpers, fostering the evolution of heterogeneous cognition. Throughout this process, participants experience an elevation in renown or material prosperity, signifying a transformation in their individual attributes. We formalize this process by introducing the concept of favor value, which is a parameter possessed by individuals and can be reciprocally contributed and received from others. Concurrent with gaming, favor value circulates among individuals, depicting diverse interactions. Ultimately, the variation in favor value contribution intensity between individuals signifies heterogeneous cognition. Furthermore, this heterogeneity in cognition influences both sides of an interaction, where those who are perceived as more respected or powerful by their counterparts may potentially gain more benefits through cooperation. Building upon this foundation, we investigate here alterations in cooperation within systems on both Watts–Strogatz small-world and regular networks following the incorporation of asymmetry. Subsequently, we examine the impact of varying the intensity of giving favor value on the cooperation trend. Lastly, we observe the distribution of favor values within the network under diverse circumstances and provide potential explanations for the results.

The rest of this paper is structured as follows. In Sec. II, we mainly describe the rules of heterogeneous cognition among individuals and the asymmetric characteristics of SDG. In Sec. III, we present the main simulation results and discussion. In Sec. IV, we conclude our work and present some future outlooks.

II. MODEL

In practical scenarios, cognitive disparities invariably emerge among individuals, attributed to factors like status inequality, economic disparities, or disapproval of each other's conduct. However, cooperative behaviors persist within this milieu. It is the very cognitive distinctions that engender unequal outcomes in cooperative efforts, with the more esteemed or influential individuals typically securing greater payoffs. The outcome of the interaction also affects individuals' cognition of their diverse neighbors all the time, contributing to intricate and pervasive asymmetric interactions in the evolutionary dynamics of society.

To describe this phenomenon, in this section, we primarily elaborate on our model from the perspective of heterogeneous cognition. In Sec. II A, we provide the details of the asymmetric characteristics of SDG, elucidating its mapping mode. Moving on to Sec. II B, we expound upon the evolution method of heterogeneous cognition, putting forward the concept of favor value. Subsequently, in Sec. II C, we introduce our novel fitness method and its corresponding individual strategy evolution approach.

A. SDG with asymmetric cost

Given the presence of heterogeneous cognition among individuals, the result of their interactions frequently exhibits asymmetry. In practical scenarios, individuals who offer greater assistance to their neighbors tend to reap more payoff in cooperative efforts, i.e., fewer losses in SDG. Consequently, we initially considered a snowdrift game characterized by asymmetric losses arising from the heterogeneous cognition among individuals. For the classical SDG, two players decide to choose either cooperate (C) or defect (D) simultaneously. The reward R or punishment P will be received by both players for mutual cooperation or defection. While one player cooperates and the other defects, the sucker's payoff S will be obtained by the cooperator and the temptation to defect T will be obtained by the defector. These payoffs satisfy the rank $T > R > S > P$, where the payoff matrix often described as $T = b$, $R = b - r/2$, $S = b - r$, and $P = 0$, with the constraint $0 \leq r \leq b$, and b is generally set to 1. When both players in the game adopt cooperation, they receive equal benefits, which is contrary to the outcomes of interactions in most real-world situations. To improve this lack, we use a parameter Λ to reflect the asymmetry degree caused by mutual assistance between individuals, while preserving the essence of the SDG. The payoff matrix for individual i while playing the game with individual j is

$$G_{ij} = \begin{pmatrix} 1 - r \cdot \Lambda_{ij} & 1 - r \\ 1 & 0 \end{pmatrix}, \tag{1}$$

where r is the cost and Λ_{ij} is the asymmetric parameter, with $0 \leq \Lambda_{ij} \leq 1$. Similar to the symmetric SDG, when one of the individual pair (i, j) chooses to cooperate, and the other defects, the cooperator bears all the losses r and the defector gets all the benefits. As i and j cooperate, they lose different costs due to the presence of Λ.

To better illustrate the asymmetric degree Λ, we introduce the concept of "favor value." It serves as a parameter inherent to individuals that can be reciprocally contributed and received from others. It denotes a specific resource each individual possesses and, to some extent, displays their distinctive characteristics. The total favor value in the network is fixed, and in the initial case, each player has a favor value of the same size, denoted by $v_{initial}$. Based on this, the asymmetric parameter Λ_{ij} from i to j is defined as

$$\Lambda_{ij} = \begin{cases} 0 & A_{ij} - A_{ji} > 1, \\ \frac{1-(A_{ij}-A_{ji})}{2} & -1 \leq A_{ij} - A_{ji} \leq 1, \\ 1 & A_{ij} - A_{ji} < -1, \end{cases} \tag{2}$$

where A_{ij} denotes the sum of favor value contributed by i to j over time, which we expect to have a linear relationship with the asymmetry degree between individuals. The range of Λ_{ij} is controlled between -1 and 1 to ensure that individuals do not have a negative income. In instances where both participants in the game select cooperation, their combined total payoff aligns with the symmetric SDG, i.e., $2 - r$. When the total favor value contributed to each other is close enough or identical, the parameter Λ gradually tends toward $1/2$, leading to the situation that the two individuals are engaged in an approximately symmetric game.

B. Evolution of heterogeneous cognition

As favor value also plays a vital role in delineating the process of inter-individual cognition formation, in our model, the mutual contribution of favor value between individuals forms heterogeneous cognition, which affects the payoff of individuals in the asymmetric game. Its specific formation and evolution process is described as follows, first, for a randomly selected node i, calculate its current favor value v_i:

$$v_i = v_{initial} + \sum_{k \in \Omega} (A_{ki} - A_{ik}), \tag{3}$$

where $v_{initial}$ denotes the initial favor value that each node has and Ω indicates the set of the neighbors of i. Then, i randomly selects its neighbor j after playing the asymmetric SDG for one time. For the single payoff sp_{ij} of i that only gets from playing the asymmetric SDG with j, we define the single fitness sf_{ij} from i to j,

$$sf_{ij} = v_i \cdot sp_{ij}. \tag{4}$$

After that, individual i hypothesizes the situation in the symmetric SDG through the current strategy and knows the potential single payoff sp'_{ij}, thus getting the potential single fitness $sf'_{ij} = v_i \cdot sp'_{ij}$ through Eq. (4). According to the sf_{ij} and sf'_{ij}, i decides to contribute favor value to j with a probability determined by the Fermi update rule,

$$p_{i \to j} = \frac{1}{1 + e^{(sf_{ij} - sf'_{ij})/k}}, \tag{5}$$

where $k > 0$ is environmental noise. It is noteworthy that ν and k represent the uncertainty to make the decision. ε is the contribution intensity, which represents the favor value that i could contribute to j in a time step. As i contributes its favor value of size ε to j, there would be the following changes: $v_i = v_i - \varepsilon$ and $v_j = v_j + \varepsilon$. Particularly, when $\varepsilon = 0$, individuals play a symmetric SDG. According to the simulation, nodes whose payoff is less than the assumed payoff will be more inclined to help others and contribute their favor value. For nodes whose payoff is greater than the assumed payoff, there is still a probability that they will continue to help others, but with the increase in the payoff, the trend of giving favor value to others will be gradually suppressed, thus reducing the occurrence of extreme cases. Also for this reason, we set a minimum boundary with a value equivalent to ε for each node's favor value, a node is restricted from contributing further when its favor value falls below this designated minimum.

C. Strategy evolution

In our model, the game strategy and favor value co-evolve within the network. Owing to the mutual contribution of favor values among individuals, we consider employing favor value as a metric for individual fitness. In particular, we define the fitness of node i as

$$f_i = v_i \cdot p_i, \tag{6}$$

where p_i denotes the accumulated payoff of i only in the current step. There is a linear relationship between individual fitness and its favor value. Node i modifies its strategy and concurrently updates by emulating a randomly selected neighbor j. The probability of adopting the strategy of the randomly selected neighbor node j is based on the Fermi update rule,

$$W_{i\leftarrow j} = \frac{1}{1 + e^{(f_i - f_j)/s}}, \quad (7)$$

where f_i and f_j are the fitness of node i and node j, respectively, and $s > 0$ represents the external noise in the decision-making process. This implies that individual strategies are shaped not only by self-interest and current resources but also by environmental factors.

To better describe our co-evolution model, in Fig. 1, we illustrate the instantaneous transformation of a network with a contribution intensity of $\varepsilon = 0.01$. In Sec. III, we will present simulation results derived from this model.

III. SIMULATION RESULTS AND DISCUSSIONS

In this section, we present the simulation method of our model, show the findings, and conduct relevant analysis. We considered two types of networks: (i) The grid lattice network with periodic boundary is a well-known example of a simple regular network in which all nodes have the same degree. Building upon the grid lattice, the Moore grid incorporates additional connecting edges between each node and its diagonal counterparts, i.e., each node is connected with eight surrounding nodes. (ii) The structure of the WS small-world network lies intermediary between a regular network and a random network, capturing the phenomenon of six degrees of separation observed in reality.[50] Hence, our simulation results are derived within the context of the Moore grid and WS small-world network, unless explicitly stated otherwise. Specifically, in the simulation process, we initially focus on examining the impact of the game cost r and contribution intensity ε on the cooperative behavior in the system. Subsequently, we analyze the temporal evolution of the strategy distribution at the microlevel within the Moore grid lattice. Finally, we investigate the distribution of favor values across various parameter pairs and provide a thorough discussion of the simulation outcomes.

The simulation is carried out in Python. At the beginning of each simulation, $N = 1296$ players are embedded into the WS small-world network or the 36×36 Moore grid with periodic boundary. In the initial stage of each simulation, the same number of cooperators and defectors is randomly distributed in the network, and their favor value is set to 1. At the same time, the environmental noise during interaction $k = s = 0.15$. Except for experiments that pay attention to evolutionary processes, all results are obtained on the iterative $T = 5000$. To eliminate the chance of experimental results and obtain more accurate results, the final results of each group of parameters were averaged over ten independent experiments.

A. Heatmaps of effect of parameter pair (r, ε) on cooperation frequency

First, we analyze the effects of SDG cost r and contribution intensity ε on cooperation evolution on different network

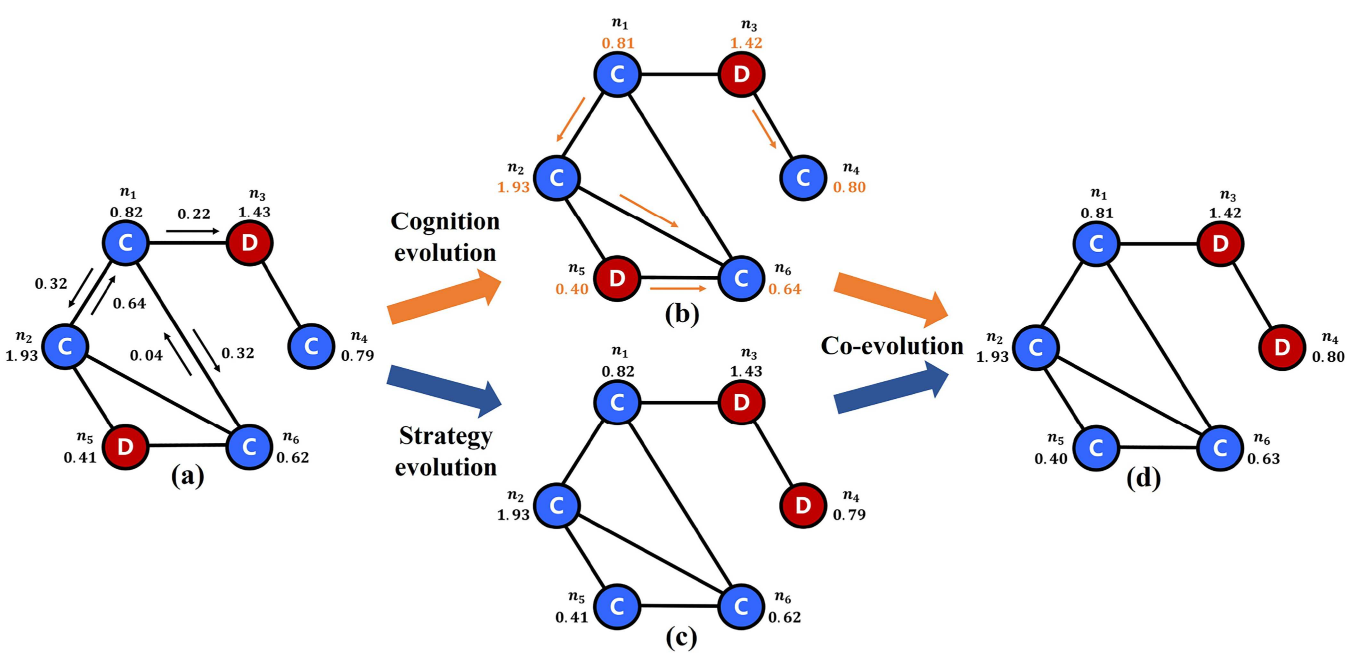
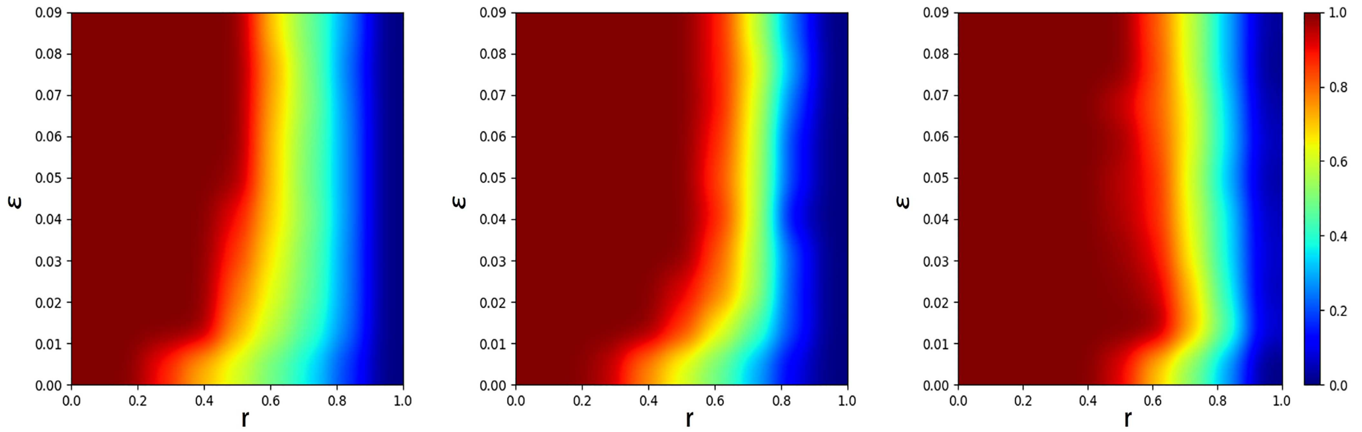


FIG. 1. An example for co-evolution of cognition and strategy. Cooperative nodes are represented in blue, while defectors are depicted in red. This process with the contribution intensity of $\varepsilon = 0.01$. (a) denotes the initial status of a network. The favor value belonging to each node is indicated adjacent to it. The black arrows elucidate the favor value income of node n_1. Nodes play the asymmetric SDG with current strategies and receive the corresponding payoff at this moment. (b) illustrates the cognition evolution on the network, with nodes contributing the favor value, depicted by the orange arrows. Simultaneously, (c) displays the strategy evolution process. (d) shows the co-evolution result that combines (b) and (c).



(a) Moore grid (b) Triangular lattice (c) WS small-world

FIG. 2. Cooperation frequency for contribution strength ε and cost r. (a)–(c) correspond to the experimental results of the model on the two-dimensional Moore grid lattice, the regular triangular lattice, and the WS small-world network. In Moore grid and regular triangular lattice, the cooperation frequency increases with the increase of ε. In the WS small-world network, the frequency of cooperation is reversed with ε. But all three are higher than for $\varepsilon = 0$ (the model is a symmetric SDG).

architectures. As a common regular network, the regular triangular lattice is also introduced in this experiment with a size of 1296 nodes and periodic boundary conditions. According to the above content, when $\varepsilon = 0$, the model is a traditional symmetric interaction, thus we can study whether the asymmetric interaction caused by heterogeneous cognition can promote cooperation behavior. The simulation results are illustrated in Fig. 2.

Figure 2(a) gives the experimental results of the model on the Moore grid. Compared to the symmetric interaction, higher cooperation frequency in the case of asymmetry is maintained for a large range of r. The results also verify the relationship between cooperation density and contribution intensity ε. When ε gradually increases from 0.01 to 0.09, the cooperation frequency is promoted to some extent. For a regular triangular lattice, the corresponding simulation results are shown in Fig. 2(b). Cooperation increases with different sizes of r when $\varepsilon \neq 0$. As ε increases from 0.01 to 0.04, there is a notable upsurge in cooperation frequency. Subsequently, as epsilon surpasses 0.04 and continues to ascend, the cooperation frequency undergoes a more gradual transformation. The simulation results on the WS small-world networks are displayed in Fig. 2(c). With a smaller ε in the designated range, the system's cooperation density experiences a significant boost. Conversely, as the contribution intensity increases, it gradually diminishes. Nevertheless, the simulations indicate that cooperation density still improves in the context of asymmetric interactions.

In the Moore grid and the triangular lattice, approximately the same change trend of cooperation is displayed. Under the same cost r, the cooperation density on the regular triangular lattice is higher than on the Moore grid. Notably, the trend of cooperation density with ε changes in WS small-world networks exhibits a reversal compared to the Moore grid and the triangular lattice. A possible explanation for this phenomenon is closely tied to their distinct network structures. The Moore grid and triangle lattice are regular networks, while the WS small-world network represents an intermediary structure between a regular and a random network.

B. Snapshots on the Moore grid lattice

Snapshots offer a comprehensive and static perspective for investigating evolutionary games on complex networks. It proves advantageous for observing the node behavior at the microscopic level. To intuitively examine the impact of complex group interactions on cooperative behavior in spatially structured populations and the variations in group cooperative behavior with changes in giving strength ε, we conducted an observational analysis of characteristic strategy snapshots in the Moore grid lattice. This exploration was carried out under different parameter pairs (r, ε), specifically $(r, \varepsilon) = (0.6, 0.03)$, $(0.6, 0.01)$, and $(0.5, 0.01)$, at time points T of 0, 2000, 4000, and 5000, respectively. The observations are shown in Fig. 3(a). The simulations, conducted with three sets of parameters, beginning with identical initial random conditions, where cooperators are represented by the color blue and defectors by the color red. The distribution of cooperators and defectors stabilizes after 2000 iterations.

The changing trend of cooperation density aligns with the results of previous experiments. With parameters set at $r = 0.5$ and $\varepsilon = 0.01$, the cooperators gradually become dominant and resist the defector's invasion. As $r = 0.6$ and $\varepsilon = 0.01$, the number of defectors rises, leading to mutual resistance of two sides and the formation of multiple clusters. As ε increases to 0.03, while maintaining r at 0.6, the frequency of cooperators further rises, with more distinct cluster features. This indicates that within the Moore grid, an increase in the contribution intensity leads to the formation of larger cooperation clusters, effectively enhancing the density of cooperators. It is widely recognized that an increased cost r inhibits the emergence of cooperation. However, a comprehensive understanding of the impact of variations in the contribution

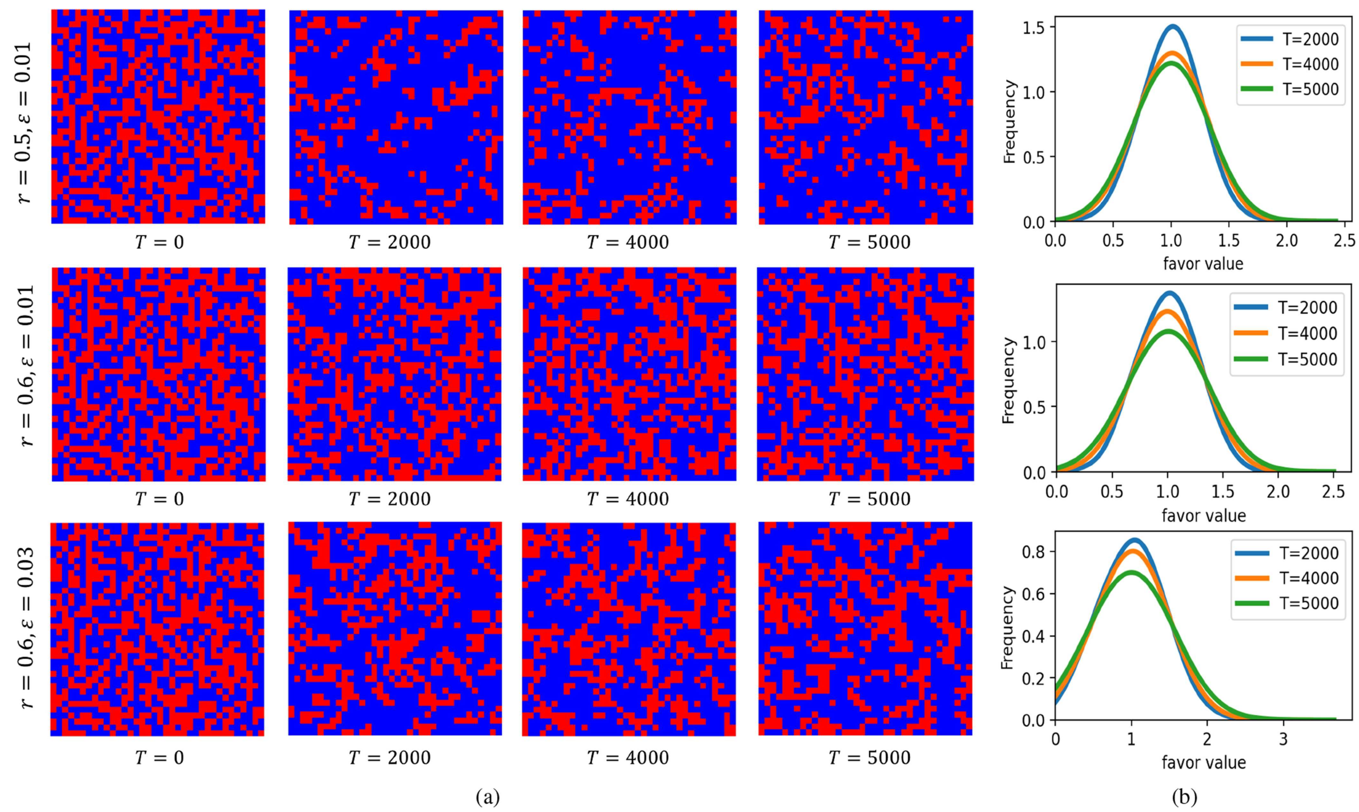


FIG. 3. Characteristic snapshots and distribution of favor value for different (r, ε) parameter pairs. (a) illustrates the characteristic snapshot of nodes' strategy under different (r, ε) parameter pairs in the Moore grid lattice. From top to bottom, the parameter pairs are set as $(0.5, 0.01)$, $(0.6, 0.01)$, $(0.6, 0.03)$, and from left to right, the time is set as $T = 0, 2000, 4000, 5000$. Blue represents cooperators, and red represents defectors. With the advance of time, the cooperators and defectors resist each other and form multiple clusters. The changing trend of cooperation density is consistent with the results of previous experiments. As the game cost r increases, the frequency of defectors increases as well. When the contribution intensity ε increases, cooperators in the lattice gradually dominate. (b) displays the distribution of favor values corresponding to different parameter pairs at various times. The blue, orange, and green lines show the distribution of favor value when T is 2000, 4000, and 5000, respectively. The distribution of favor values shows a generally similar trend under different conditions.

intensity on cooperative behavior necessitates an exploration of the distribution of favor values within the system.

Figure 3(b) illustrates the distribution of favor value corresponding to different (r, ε) parameter pairs at various time points. The blue, orange, and green lines show the distribution of favor value when T is 2000, 4000, and 5000, respectively. It can be observed that the flow trend in favor value becomes more pronounced over time. Despite the initial setting, where individuals with a favor value equal to 1 constitute the majority, this proportion diminishes over time, and the range of favor values also expands. This indicates a gradual widening of the gap in favor values in the network. At a fixed time, the evolution of favor values exhibited a nearly identical trend in both cases when $\varepsilon = 0.01$. With $r = 0.6$, a higher ε leads to the emergence of more individuals with favor values near the minimum limit. The kurtosis of favor value decreases and the positive skewness increases. Additionally, the maximum favor value attainable by individuals is higher under elevated ε.

In asymmetric interactions, both partners in cooperation exhibit a tendency to contribute higher favor values, aiming to maximize their respective benefits. Furthermore, there exists a probability that defectors may receive favor values from their neighbors. As evolution progresses, individuals possessing lower favor values may show a tendency to imitate strategies from those with higher favor values in their connections, given the associated fitness advantages. Consequently, clusters of cooperators or defectors emerge, gravitating around individuals with elevated favor values. Simultaneously, the escalating contribution intensity aligns with individual proclivities, fostering a more obvious flow of favor value. Meanwhile, with an increase in the cost r, individuals with lower favor values become less inclined toward cooperation, displaying a greater propensity for betrayal. This leads to a reduction in cooperation frequency and alterations in cluster dynamics.

C. Evolution of the distribution of favor value over time

Based on the aforementioned simulation results, the temporal evolution of favor values exhibits a regular pattern. To further

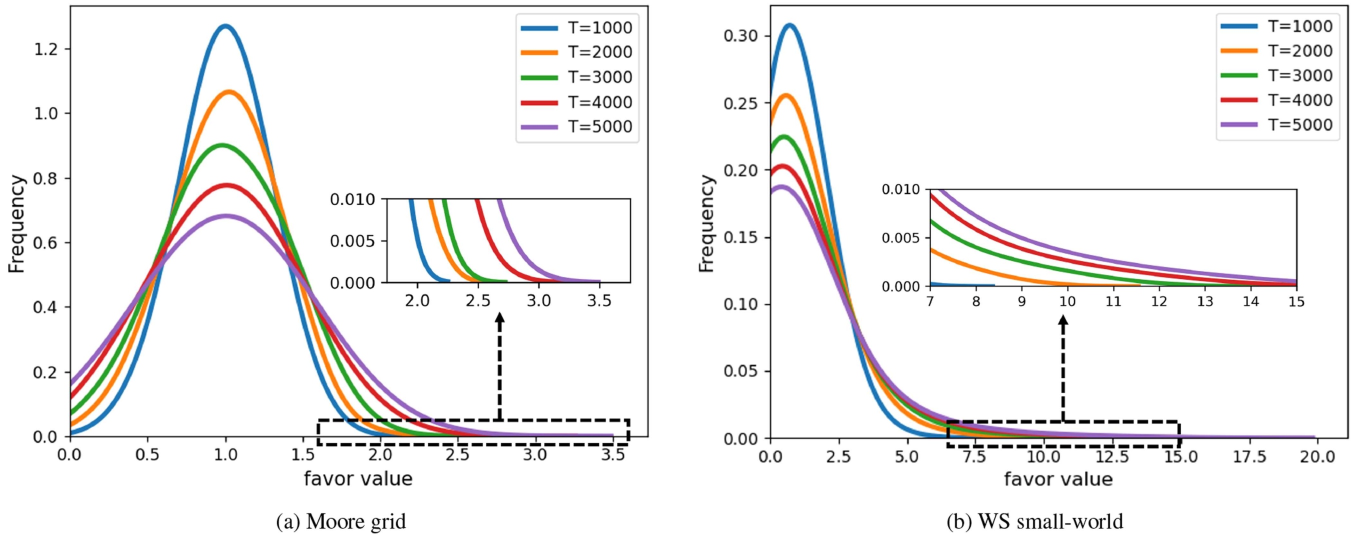


FIG. 4. Distribution of favor value over time with fixed parameters. (a) and (b) display the situation in the Moore grid and WS, respectively. The simulation is performed with a parameter pair of $(r, \varepsilon) = (0.8, 0.01)$ and observed every 1000 time steps until $T = 5000$. Over time, the trend in favor value dynamics intensifies, and the distribution of favor values in the WS network exhibits stronger extremity compared to that of the Moore grid.

investigate the influence of distinct network architectures on the favor value evolution, we examine the favor value distribution for a specific parameter pair within both the Moore grid and the WS small-world network over time. The simulations were performed with a parameter pair of $(r, \varepsilon) = (0.8, 0.01)$. We extend the time step to $T = 5000$ and observe the distribution of favor value every 1000 time steps. These observations are illustrated in Fig. 4. Given that we set the minimum favor value of each individual, the distribution features a left boundary equivalent to $\varepsilon = 0.01$.

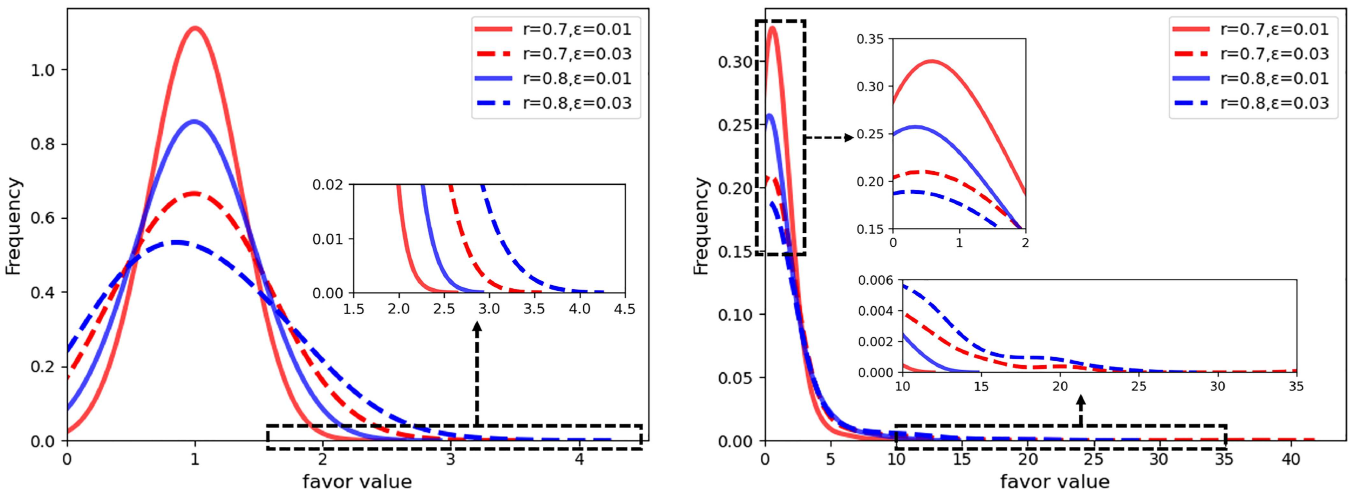
Figure 4(a) illustrates the temporal evolution of the favor value distribution in the Moore grid, consistent with observed trends from previous experiments in Sec. III B. Over time, while individuals with a favor value of around 1 continue to constitute the majority, their proportion is gradually diminishing. As T ranges from 1000 to 5000, the modal peak of the distribution exhibits a decline, diminishing from above 1.2 to approximately 0.7. Concurrently, the maximum favor value that an individual can attain is rising. The right tail of the distribution expands from 2.25 to 3.5. The decreasing kurtosis and the increasing positive skewness within the data further affirm this. Furthermore, as the cooperation behavior stabilizes, the rate of favor value evolution diminishes. Figure 4(b) displays the situation in the WS. Similar variations in statistical characteristics over time, i.e., decreasing kurtosis and rate of evolution, increasing positive skewness, are observed as mentioned above. The mode of the distribution decreased from 0.31 to 0.19, and concurrently, the right boundary expanded from 8.3 to 20. This indicates an ongoing flow of favor values in the WS network as well.

However, for an equivalent time step, the trend in favor value dynamics is more pronounced under identical contribution intensity in the WS network compared to the Moore grid. Significantly lower kurtosis and substantially higher positive skewness validate this. This phenomenon is also evident in Fig. 4, where the favor value distribution exhibits a stronger dispersion in the WS network. Furthermore, there is a notable leftward shift in the median favor value, accompanied by an increasing extreme favor value on the right side indicating that the favor value of quite a few nodes reaches the lowest, further corroborating the extreme distributions. We speculate that distinct network structures contribute to more extreme distributions on WS. It is known that the system's cooperation density achieves stability at $T = 2000$ in the preceding simulation. However, the exchange of favor values persists beyond this point. This strengthens the asymmetric degree among nodes, thereby establishing a relatively stable state of mutual cooperation or defect.

D. Effect of different parameter pairs on the distribution of favor value

Given that the evolution of cooperation varies across distinct network structures with changing parameters, and asymmetric interaction promotes cooperation in the system, following up the evolution of favor values under different parameter pairs is necessary. We maintain a constant time step and investigate the impact of various parameter pairs (r, ε) on the distribution of favor value. The simulation results are shown in Fig. 5. The blue and red lines correspond to scenarios with game costs $r = 0.8$ and $r = 0.7$, respectively. The solid and dotted lines denote cases where the contribution intensity is $\varepsilon = 0.01$ and $\varepsilon = 0.03$, respectively.

Figure 5(a) depicts the simulation results in the Moore grid. For $r = 0.7$ and $\varepsilon = 0.01$, the modal peak in the favor value distribution attains 1.12, and the right boundary is 2.40. With an increase in ε to 0.03, the modal peak diminishes to 0.7, and the right boundary rises to 3.15. Simultaneously, the kurtosis decreases, while positive skewness and dispersion exhibit an increase. With ε held constant at 0.01,



(a) Moore grid

(b) WS small-world

FIG. 5. Distribution of favor value over with different parameter pairs. (a) and (b) show the situation in the Moore grid and WS, respectively. The blue and red lines show the situation of the game cost $r = 0.8$ and $r = 0.7$. The solid and dotted lines represent cases where giving strength $\varepsilon = 0.01$ and $\varepsilon = 0.03$, respectively. When T and r are fixed, a larger ε will make the flow trend of favor value more obvious. In Moore grid, at a higher r, individuals are more willing to get more benefits by giving help, and the increase of ε makes this willingness stronger. However, in WS, when r is relatively high, as the ε increases, even if the interaction between individuals reduces the number of individuals with a favor value around 1, no individual with a greater favor value appears.

the modal peak decreased to 0.87, and the right boundary increased to 2.77 as r escalated from 0.7 to 0.8. Concurrently, positive skewness and the degree of dispersion also increased, while kurtosis exhibited a decrease. All alterations in the favor value distribution induced by variations in r or ε during the simulation adhere to the aforementioned two principles. The results in WS are presented in Fig. 5(b). The distribution exhibits stronger extremeness compared to the situation in the Moore grid. When $r = 0.7$ and $\varepsilon = 0.01$, the modal peak of the distribution is 0.33, and the right boundary is 13. As ε increases to 0.03, its modal peak decreases to 0.21, while the right boundary increases significantly to 42. Its kurtosis decreases, while positive skewness and dispersion increase. Meanwhile, a similar scenario unfolds when ε is held constant and r is increased. With $\varepsilon = 0.01$, as r rises from 0.7 to 0.8, the modal peak of the distribution decreases from 0.33 to 0.26, and the kurtosis diminishes. The right boundary of the distribution ascends from 12 to 15, accompanied by an increase in positive skewness and dispersion.

In the Moore grid, at a higher r, individuals are more willing to get more benefits by giving help, and the increase of ε makes this willingness stronger. However, in WS, as $r = 0.8$, when the ε increases, even if the asymmetric interaction reduces the frequency of individuals with a favor value around 1, the maximum favor value that an individual can have does not grow much. Even the maximum favor value when $r = 0.7$, $\varepsilon = 0.03$ is much bigger than the situation when $r = 0.8$ and $\varepsilon = 0.03$, which does not match the situation in the Moore grid. A possible explanation for this phenomenon is rooted in the distinct network characteristics. As the contribution intensity ε increases, individuals in WS are more concerned about the consequences of providing assistance. Consequently, they exhibit a more cautious approach to prevent substantial losses in favor value and payoff resulting from defecting. This is in contrast to the dynamics observed in the Moore grid. The variation trend of the cooperation frequency with changes in ε is thus resolved in both regular and WS small-world networks.

IV. CONCLUSION AND OUTLOOK

In this paper, we studied a snowdrift game with asymmetric costs arising from heterogeneous cognition. We delineate the generation and evolution of cognition, defining fitness based on this attribute. To realize the co-evolution of cognition and cooperation, we introduced the favor value. Specifically, individuals are endowed with favor value as an attribute, and they establish heterogeneous cognition by contributing or obtaining favor value with a fixed contribution intensity each time. This cognitive relationship leads to unequal payoffs in the interactions between individuals in the game. We conducted simulations to examine the impact of SDG cost (r) and contribution intensity (ε) on the system's cooperation behavior and unearthed distinct patterns of cooperation evolution on both WS and regular networks. Following this, we examined the strategy distribution at the microlevel on the Moore grid lattice to identify the internal factors influencing cooperation. The presence of asymmetry enables both partners who cooperate to attain higher fitness, resist defection behaviors, and consequently, foster the emergence of cooperation. To further observe the role of favor value in evolution, we discuss its distribution in the system over time and for different parameter pairs (r, ε). We find that the contribution of favor value serves to stabilize the asymmetry within the system, consequently contributing to the stability of cooperative outcomes. In the context of WS, owing to its inherent network structure, the distribution of favor values exhibits stronger extremity and dispersion. Simultaneously, with an increase in donation intensity, individuals in the WS demonstrate heightened concern regarding the repercussions of extending aid. Consequently, they adopt a more cautious stance to forestall significant losses in favor value and reciprocation resulting from defection. This is in contrast to the dynamics observed in the Moore grid. Such disparities offer a potential explanation for the variations in cooperation within the network.

The asymmetric properties proposed in our study stem from the behavior of individuals contributing certain properties to each other. This dynamic allows for a fluctuation in the asymmetry degree among individuals. However, the impact of diverse manifestations of heterogeneous cognition on the asymmetry degree, such as nonlinear mapping, and whether they yield similar outcomes or induce distinct phenomena, remains a focal point for our future research. We anticipate that this investigation will inspire future studies aimed at probing the effects of properties that can flexibly represent individual characteristics in the systems. It encourages the exploration of structured groups through empirical validation experiments and endeavors to contribute to their further development for addressing social dilemmas.

ACKNOWLEDGMENTS

This research was supported by the National Natural Science Foundation of China (NNSFC) under Grant No. 62206230, the Natural Science Foundation of Chongqing under Grant No. CSTB2023NSCQ-MSX0064, and the Humanities and Social Science Fund of Ministry of Education of the People's Republic of China under Grant No. 21YJCZH028.

AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Yuxuan Jing: Conceptualization (equal); Data curation (equal); Formal analysis (equal); Methodology (lead); Visualization (equal); Writing – original draft (equal). **Songlin Han:** Investigation (equal); Methodology (equal); Resources (equal). **Minyu Feng:** Conceptualization (equal); Funding acquisition (lead); Investigation (equal); Methodology (equal); Supervision (equal); Writing – original draft (equal). **Jürgen Kurths:** Project administration (equal); Supervision (equal); Writing – original draft (lead).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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