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An evolutionary game with conformists and profiteers regarding the memory mechanism

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A B S T R A C T

Network evolutionary game theory provides a new perspective on how cooperative behaviors emerge in the real world and has been widely investigated. However, most studies assume that players are profiteers and ignore the existence of conformists and that players have memories, which are crucial when people make decisions. In this study, we study a memory-based snowdrift game occurring on networks and propose two strategy-updating rules based on profiteers and conformists while considering the historical strategy, memory strength, payoff information and memory length to discuss the emergence and maintenance of cooperation behaviors. In contrast to previous studies, we introduce the player's degree of cooperation to continuousize player payoffs and we consider it when defining the player's strategy-updating rules. In simulations, we show the evolution of the frequency of cooperation as time progresses and investigate the effects of the payoff parameter, memory strength, memory length and conformist ratio on the frequency of cooperation, and further validate the robustness of our model using different network sizes. Our results show that the memory strength, memory length, and conformist ratio can facilitate the cooperation level of the network over a large parameter area, and that the size of the network has almost no effect on the model, which shows the robustness of our model. Our work may elucidate the study of evolutionary games with conformists and memory effects.

1. Introduction

The emergence of group cooperation behaviors has been a major issue for researchers in various academic fields [1,2]. There have always been cooperators both in nature and human societies. However, a selfish individual has no reason to give up a defection strategy that could obtain more benefits than that obtained by cooperating with others. This problem has always troubled humans. The emergence of network evolutionary game theory has provided a theoretical and effective framework for studying this problem. Within this framework, the prisoner's dilemma game (PDG) [3–5], snowdrift game (SG) [6–8], and stag hunt game (SHG) [9–11] have been deeply studied. These studies are conducted on complex networks, such as square-lattice networks [12], small-world networks [13,14] and scale-free networks [15,16]. Complex networks have rapidly progressed, with various novel network models having been proposed in the recent years [17–19]. In networks, each node denotes a player and each edge represents their interaction relationship. Each player plays a specific type of game with his/her neighbors and updates his/her strategy according to his/her payoff. In general, the game model and network topology strongly influence the evolution of cooperation. Conversely, to reduce

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the frequency of defectors in a population, the following five renowned mechanisms [20] promoting cooperation have been proposed: kin selection [21], direct reciprocity [22], indirect reciprocity [23], group selection [24], and network reciprocity [25–27]. Other proposed mechanisms, such as reward [28], reputation [29], punishment [30], persistence [31] and so on [32–35], have proved to be effective in increasing the frequency of cooperators.

Previous studies concerning the evolution of cooperation have largely assumed that players in the network have the same payoffs for the same strategy pairs. However, in reality, a player playing a game against two players with the same strategy may obtain different payoffs, i.e., the payoff obtained by the player is related to the strategy between players and the player's own attributes. Therefore, we introduce the degree of cooperation of players into our model to continuousize the player payoffs in this study. Within the degree of cooperation of players, the same strategy pair in the network will have different payoffs because the degree of cooperation of players between interactions is different. Moreover, in the traditional network evolutionary game model, the probability of the strategy changes during the next step is determined using the player's performance in the previous step. In other words, players are considered to be shortsighted and forgetful. When people make an important decision, they generally consider the current situation and their historical experience. Therefore, the influence of memory should be considered because historical memory is crucial in evolutionary games. Therefore, numerous studies have been conducted. For example, Liu et al. introduced a preferential parameter to describe the nonlinear correlation between selection probability, strategy persistence, and the memory length of individuals into evolutionary games [36]. Shu et al. proposed a mechanism of conformity with memory, whose core lies on two facts [37]. Zhao et al. studied a memory-based weak PDG in interdependent networks [38]. Dong et al. studied a memory-based SHG on regular lattices [39]. Lu et al. investigated the influence of the memory effect on the evolution of cooperation in a spatial PDG [40].

Over the past few decades, researchers have noticed that profiteers and conformists are crucial in social dynamics [26]. Players learn the most successful strategies and choose the most popular strategies, and there are several conformists in real life. For example, we have noticed that when consumers shop online, they tend to buy mass-market goods. A person's point of view is often greatly influenced by most people around him or her. In addition, there is a lot of research on conformists and great results have been achieved. For instance, Niu et al. explored the effects of the rational conformity behavior on the evolution of cooperation in PDG [41]. Yang et al. investigated the propagation of cooperation in a conformity-driven dynamic social network where an agent adjusts a social tie sometimes in accordance with a rival's strategy's popularity [42]. Zhang et al. studied the effects of imitation, aspiration, and conformity-driven dynamics on the evolution of cooperation in a square lattice with periodic boundary conditions [43]. Pi et al. investigated the effect of conformists on the evolution of cooperation in multigames [44]. In a study [45], researchers studied the effect of different proportions of various types of network conformists on the proportion of network cooperators.

In view of the aforementioned points, a new and practical model is proposed to discuss the evolution of cooperation, where each player is located on the node of the network and is assigned different degrees of cooperation d to continuousize the player's payoffs, with the same memory strength c and the same size of memory length T to provide players with memory features. We present a fully new decision-making mechanism and combine it with traditional strategy-updating rules for players updating strategies. Different from previous works related to memory mechanisms [46], considering the close relationship between the time dimension and decision-making behavior, the new mechanism records the strategies used in the player's history instead of, the payoffs gained by the player's history. Furthermore, the noise impact on different players should be different, which is contrary to the fixed noise factor κ in the traditional Fermi function. Thus, we define the difference in the degree of cooperation as the similarity between players and allow it to replace the traditional noise factor κ, which is more in line with reality. Therefore, we propose two novel strategy-updating rules based on profiteers and conformists regarding the players' memory mechanism. Using the selected strategy, the player plays the SG with his/her immediate neighbors on square-lattice networks and small-world networks. We examine whether the historical strategy memory mechanism can facilitate cooperation behaviors using different memory strengths, payoff parameters, memory lengths, and conformist ratios. The results obtained via our simulations show that the cooperation behavior of the network can be promoted by the memory strength, memory length, and conformist ratio over a large parameter area.

This study is organized as follows. In Section 2, we describe our model: players' payoff calculation and strategy-updating rules for different types of players. In Section 3, we describe our simulation methods, report, and elaborate our results. Finally, we summarize the conclusion and describe the outlook in Section 4.

2. Model

In this section, we mainly illustrate the details of our game model based on players with memory mechanisms, the calculation of a player's payoff, and strategy-updating rules based on profiteers and conformists.

2.1. The game model and calculation of a player's payoff

First, we briefly describe the original SG model. Consider two people driving in opposite directions on a windy and snowy night and are blocked by a huge snowdrift. Both drivers can get out of the car to shovel or stay in the car, which we call a cooperator (C) or a defector (D), respectively, under one negotiation. Suppose the cost of shoveling this snowdrift to clear the road is q, and the benefit to each person of clearing the road is b, we have $b > q$. If both drivers choose to shovel the snowdrift (C), then they both gain benefit b of getting back home while sharing the labor q of shoveling, i.e., both get payoff $b-q/2$. If one driver shovels the snowdrift (C) while the other driver stays in the car (D), although both drivers can go home, the defector escapes the labor and gets a perfect payoff b, whereas the payoff of the cooperator is $b-q$. If both drivers choose to stay in the car and not shovel the snowdrift (D), they will still be trapped by the snowdrift and cannot go home; therefore, their benefits are 0. Without losing generality, $b-q/2$ is usually set to 1 so that the evolutionary behavior of the SG can be investigated with a single parameter, $r=q/2=q/(2b-q)$. Therefore, the payoff matrix of the SG is expressed as follows:

$$M=\begin{pmatrix} 1 & 1-r \\ 1+r & 0 \end{pmatrix} \tag{1}$$

where $0<r\leq 1$ is a regulable parameter. Generally, cooperation will dominate when $r\to 0$, and defection will dominate when $r\to 1$.

Herein, we illustrate the evolutionary process of the memory-based SG. We assume that N players exist in a given network, in which each node represents a player. In every round, each player in the evolutionary game can only choose to be a cooperator (C) or a defector (D) at each strategy-updating step. Specifically, in this study, we make a more realistic assumption that each player can choose his/her strategy from a continuous set of strategies at the beginning. In detail, each player in the structured population can choose a strategy value of d, which means the degree of propensity to cooperate and falls within the interval [0, 1]. Every individual is randomly assigned a strategy value of d, which follows a uniform distribution and determines the player's strategy. In particular, we specify that a higher value of d means a higher cooperativeness level in the spatial population. Notably, $d=1$ and $d=0$ correspond to fully cooperative and fully defective strategies, respectively. Furthermore, $0.5<d<1$ indicates a predominantly cooperative strategy and $0<d<0.5$ represents a predominantly defective strategy.

Without losing generality, we study the pairwise evolutionary game in this study. In a pairwise interaction, let player i and player j having continuous strategies d_i and d_j play a game, respectively. Then, if they are both cooperators, they get d_id_j as the benefit of mutual cooperation. If player i is a defector and player j is a cooperator, then player i is given $(1+r)(1-d_i)d_j$ and player j obtains $1-r$. Moreover, if the two players are both defectors, they get nothing. Each player plays the game with all his/her neighbors to acquire his/her payoff Π through the aforementioned rules. Thus, the payoff of player i can be given as follows:

$$\Pi_i=\begin{cases} \sum_{j\in\Omega,s_j=C} d_id_j+\sum_{j\in\Omega,s_j=D}(1-r), & \text{if } s_i=C \\ \sum_{j\in\Omega,s_j=C}(1+r)(1-d_i)\,d_j, & \text{if } s_i=D \end{cases} \tag{2}$$

where Ω indicates the neighbor player set of player i and s_i denotes the strategy of player i.

2.2. Strategy evolution

In this subsection, we describe the strategy-updating rules based on different types of players. In the proposed model, we consider that there are two types of players: profiteers and conformists, which are evenly mixed in the network. The players synchronously update their strategy, such that all players' strategies will be updated during each round of the game.

In reality, players are not always the same and would update their strategies via different methods after playing a round of the game with their neighbors, which affords adopting different strategy-updating rules. Therefore, we define two different player types: profiteers and conformists. Profiteers with memory mechanisms are more inclined to choose the strategies of high-payoff players, i.e., a player i (profiteer) adopts the strategy s_y from player y (randomly chosen from neighbors) with a probability that depends upon the payoff difference and the set of strategies he/she has used in the past is expressed as follows:

$$P(s_x\leftarrow s_y)=c\,\frac{\sum_{i=1}^{T}\delta\left(S_{x,T}(i),s_y\right)}{T}+(1-c)\frac{1}{1+e^{(\Pi_x-\Pi_y)/|d_x-d_y|}} \tag{3}$$

where c indicates memory strength, $S_{x,T}$ represents player $x's$ strategy in the past T game rounds, and $\delta\left(S_{x,T}(i),s_y\right)$ can be denoted as follows:

$$\delta\left(S_{x,T}(i),s_y\right)=\begin{cases}1, & S_{x,T}(i)=s_y\\ 0, & S_{x,T}(i)\neq s_y\end{cases} \tag{4}$$

where Eq. (4) is a Dirac function. The player's memory length will be regarded as t when $t<T$ because the evolution time at this point has not yet reached T.

We consider Eq. (3) as the memory-based profiteer-driven strategy-updating rule. Additionally, the degree of propensity to cooperate d_y of player y will be adopted by player x when player x has adopted the strategy of player y.

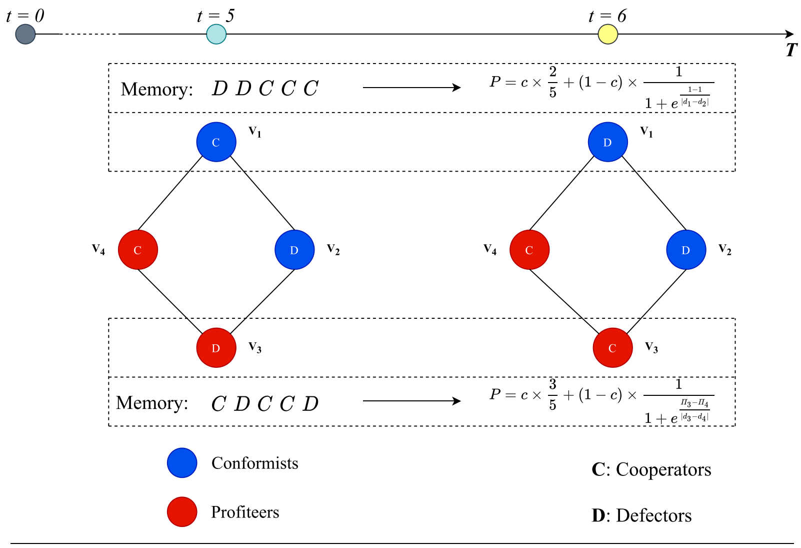


Fig. 1. An example of the model. This figure shows an evolutionary game process with conformists and profiteers considering the memory mechanism, where the blue and red nodes represent conformists and profiteers, respectively, whereas C and D represent cooperators and defectors, respectively. At $t = 5$, profiteer V_3 adopts strategy C from player V_4 (randomly chosen from neighbors) with the probability given by Eq. (3). Because the strategies V_3 used in the previous five steps (the memory of V_3) are C, D, C, C, and D, the probability that the strategy of V_3 changes to C in the next step is $P = c \times \frac{3}{5} + (1-c) \times \frac{1}{1+e^{\frac{\Pi_3-\Pi_4}{|d_3-d_4|}}}$, where c indicates memory strength, d_i represents the degree of propensity to cooperate of player i, and Π_i denotes the payoff of player i. Conformist V_1 changes his/her current strategy with the probability given by Eq. (5). Because the strategies V_1 used in the previous five steps (the memory of V_1) are D, D, C, C, and C, the probability that the strategy of V_1 changes to D in the next step is $P = c \times \frac{2}{5} + (1-c) \times \frac{1}{1+e^{\frac{1-1}{|d_1-d_2|}}}$. All players will update their strategies at each step based on the player's own type.

Conformists with memory mechanisms are more likely to select the strategies that have a high number of users in the neighborhood, namely player i (conformist) changes his/her strategy with a probability that depends upon the number of strategies used among his/her neighbors and the set of strategies he/she has used in the past is given as follows:

$$P(s_x change) = c\frac{\sum_{i=1}^{T}\delta\left(S_{x,T}(i), s_y\right)}{T} + (1-c)\frac{1}{1+e^{(N_{sx}-k_h)/|d_x-d_y|}} \tag{5}$$

where N_{s_x} is the number of players adopting strategy s_x within the interaction range of player x, k_h is one-half the degree of player x, and s_y is the anti-strategy of s_x at this time. Analogously, we propose the memory-based conformist-driven strategy-updating rule in Eq. (5). In addition, the degree of propensity to cooperate d_x of player x will change to $1 - d_x$ when player x has changed his/her strategy.

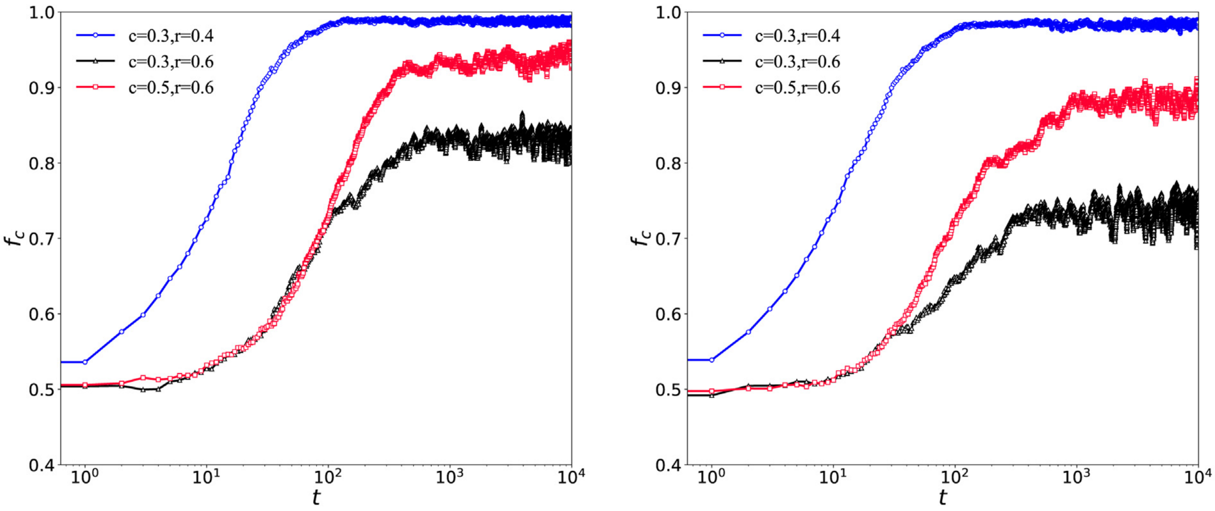
This ends the modeling section. In brief, we first introduced our SG model, then described the calculation of players' payoffs after the introduction of the degree of player cooperation and finally proposed two different strategy-updating rules based on profiteers and conformists with memory mechanisms to overcome the shortcomings reported in some previous researches. Fig. 1 depicts an example of our model. In the following section, we will illustrate the simulation methods and present our results.

3. Methods and simulation results

In this section, for confirming the previous theory, we describe the simulation methods and present the results obtained through simulations along with their analyses. We analyze the evolution of cooperation frequency over time under different parameter pairs; then, observe the effects of memory strength and payoff parameter on cooperation frequency. Subsequently, we study the influences of memory strength, memory length, and conformist ratio on the cooperation frequency and discuss the strategy distribution on the square-lattice network at the micro level during the evolution process. Finally, we investigate the influence of network size on cooperation frequency to validate the robustness of our model.

3.1. Methods

At the beginning of each simulation, unless otherwise specified, $N = 1024$ players are embedded into the WS network or a 32×32 square-lattice network, where WS is generated by regular graphs and randomized reconnection, which



(a) WS (b) RE

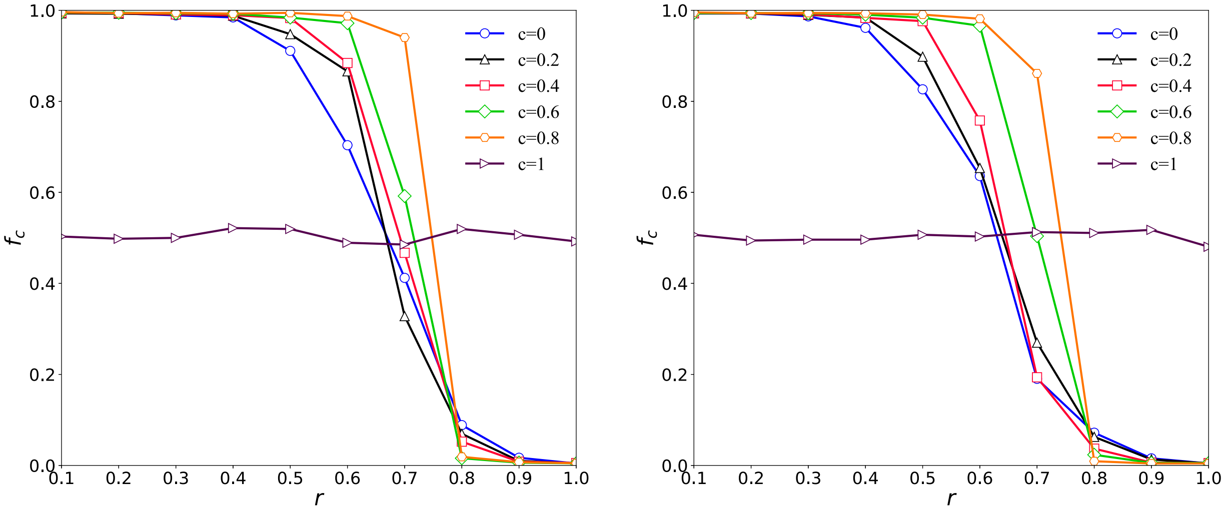
Fig. 2. Plots of cooperation frequency against evolutionary time. WS is generated by regular graphs and randomized reconnection, where the parameters are set as the number of players $N = 10^4$, the random reconnection probability $p = 0.15$, and $K = 4$, which indicates that each player is connected to two players (his/her left and right neighbors). RE is generated by a square-lattice network with 100 × 100 players. The parameter pair (c, r) is set to (0.3, 0.4), (0.3, 0.6), and (0.5, 0.6), with different last and first numbers for contrast and the memory length T is restricted to 5. Subplots (a) and (b) show the cooperation frequency against evolutionary time on the WS network and regular network, respectively, where the x-axis and y-axis are set as the time and the cooperation frequency, respectively. Each x-axis range is set as [0, 10^4], and each y-axis range is set as [0.4, 1]. We observe each network until $t = 10^4$ so that the cooperation frequency becomes stable. Regardless of the parameter pair (c, r), the cooperation frequency increases with evolutionary time and eventually reaches a stable value.

was proposed by Watts and Strogatz in 1998 [47]. Initially, a cyclic nearest-neighbor coupling network containing N nodes is given, where each node is connected to $K/2$ nodes (its left and right neighbors), and K is an even number. Then, each of the original edges in the network is reconnected randomly with probability p, i.e., one endpoint of each edge remains unchanged and the other endpoint is considered to be a randomly chosen node in the network, which specifies that there must be no multiple edges or self-loops. To produce the WS network, we mainly employ the function *watts_strogatz_graph*() of networkx in Python. The initial type of player is randomly selected from the space T = {P, C}, where P, C denotes the profiteer and the conformist respectively, and the initial degree of propensity to cooperate of player d is uniformly distributed in [0, 1], i.e., the initial strategy of the player is randomly selected from the space S = {C, D}, where C, D indicates the cooperator and the defector, respectively. Furthermore, a strategy update is implemented through the roulette algorithm. In detail, there is a probability p for each player to imitate the strategy and degree of propensity to cooperate of neighbor player y (if the player is a profiteer, the probability p is calculated using Eq. (3)) or change his/her current strategy and degree of propensity to cooperate (if the player is a conformist, the probability p is calculated by Eq. (5)). We generate a random number that follows a uniform distribution between [0, 1] and then compare it to probability p. If it is larger than p, the aforementioned behavior will not occur; otherwise, it will. In this study, we implement the evolution of cooperation in a simulation with a length T = 10000 steps. Moreover, to avoid additional disturbances, the final results were averaged for 10 independent simulations for each set of parameter values to ensure a suitable accuracy.

3.2. Evolution of cooperation frequency over time

For a population, the cooperation level is one of the most concerning indicators for people and it is commonly characterized via cooperation frequency f_c, denoting the fraction of cooperators. Therefore, to explore the evolutionary relationship of cooperation frequency f_c with respect to time t under different memory strengths and payoff parameters, we must control three parameters: c, r, and t, where c is the memory strength of player, r is the parameter of our game model (as shown in Eq. (1)) and t is set as a large number to obtain stationary results. In the following simulation, we select a memory length of $T = 5$ and three pairs of (c, r): (0.3, 0.4), (0.3, 0.6), and (0.5, 0.6), with different last and first numbers for contrast to investigate the evolution of cooperation frequency as time progresses.

For better stationary results by acquiring data for a sufficiently long time, we consider t be 10^4 and record the cooperation frequency of each parameter pair (c, r) as time progresses in a half-log coordinate. We use the half-log coordinate to better illustrate the ascent stage and the stationary process of the plots. The results of cooperation frequency f_c varying with time t are shown in Fig. 2.



(a) WS (b) RE

Fig. 3. Plots of cooperation frequency against payoff parameter. WS is generated by regular graphs and randomized reconnection, where the parameters are set as the number of players $N = 1024$, the random reconnection probability $p = 0.15$, and $K = 4$, which indicates that each player is connected to two players (his/her left and right neighbors). RE is generated by a square-lattice network with 32×32 players. We set the memory strength $c = 0, 0.2, 0.4, 0.6, 0.8$, and 1, and the memory length T is restricted to 5 in both subplots. Setting the evolution time $t = 10^4$, we obtain each cooperation frequency by averaging the cooperators proportion in the last 1000 rounds, and each data point is averaged using the cooperation frequencies of 10 independent simulations. Subplots (a) and (b) show the cooperation frequency against payoff parameter under different memory strengths on the WS network and regular network, respectively, where the x-axis and y-axis are set as the payoff parameter and the cooperation frequency, respectively. Each x-axis range is set as [0.1, 1], and each y-axis range is set as [0, 1]. We observe each network until $t = 10^4$ so that the cooperation frequency becomes stable. Except for $c = 1$, both networks exhibit that a larger value of r leads to a smaller cooperation frequency.

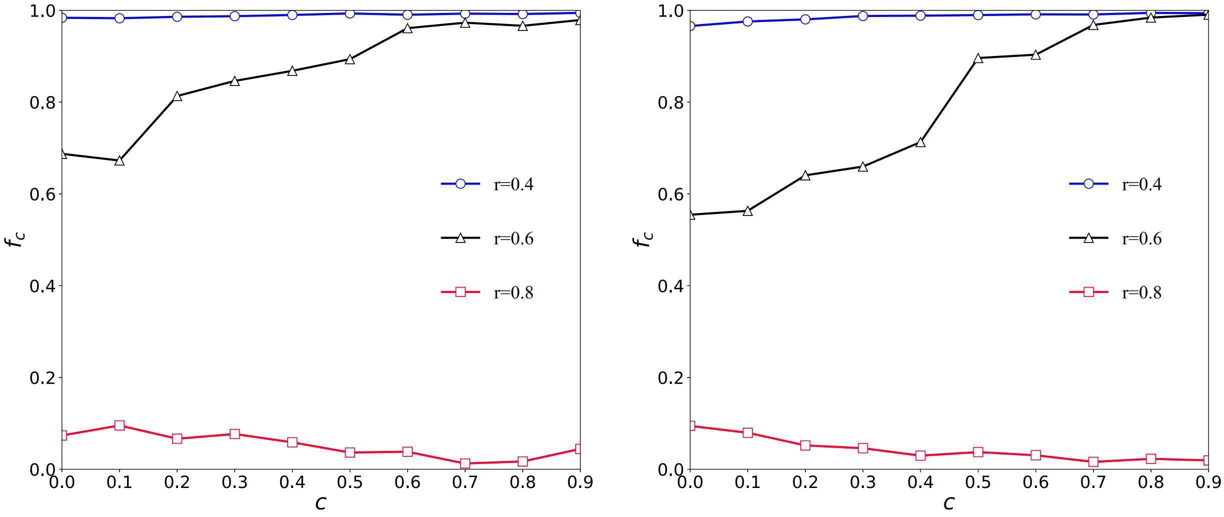
As we can clearly see, the cooperation frequency f_c in the network will become increasingly higher during the evolution with different parameter pairs (c, r). Furthermore, the results show that the stationarity of the cooperation frequency exists, all three parameter pairs (c, r) finally approach stationarity after a short time (approximately $t = 10^2$) and they all steadily float within bounds. In detail, with the same $c = 0.3$, $r = 0.4$ marked by blue circles approaches a higher value and fluctuates considerably less than $r = 0.6$ marked by black triangles. After $t = 10^2$, with the same $r = 0.6$, $c = 0.5$ marked by red squares approaches a higher value than $c = 0.3$ marked by black triangles. In other words, the higher c and lower r yield a higher stationary value f_c. Furthermore, Figs. 2(a) and 2(b) show that the stationary value of cooperation frequency is higher on the WS network than that on the RE network except for the parameter pair (c, r) = (0.3, 0.4). In addition, although the cooperation frequency evolution curves of (c, r) = (0.5, 0.6) and (c, r) = (0.3, 0.6) on the WS and RE networks are very similar in the near future, the cooperation frequency of (c, r) = (0.5, 0.6) is considerably greater than that of (c, r) = (0.3, 0.6), appearing earlier on the RE network than that on the WS network.

Subsequently, we analyze the possible reasons for the results of Fig. 2. As is well known, the cooperators will have a less payoff when r increases, and defection might be a better strategy. Thus, we deduce that a smaller r will afford a higher frequency of cooperation under the condition of the same c. Fig. 2 depicts that the effect of memory strength c is remarkable. The cooperation frequency f_c will be improved even if the memory strength c only increases slightly. Thus, the introduction of the memory effect into the SG is beneficial to the emergence of cooperation in the WS and square-lattice networks.

3.3. Effect of memory strength and payoff parameter on the cooperation frequency

As we previously described, memory strength c will have an influence on the probability that the player will update his/her strategy and affect cooperation behavior. To observe the cooperation frequency at equilibrium, the cooperation frequency f_c as a function of payoff parameter r under different memory strengths c is given in Fig. 3.

When $c = 0$, it means players only focus on the payoff difference or number of strategies, which is called being memory free. As shown in both Figs. 3(a) and 3(b), cooperators will dominate the network when payoff parameter r is small, except when $c = 1$, which will be described separately later in this study. Except for the situation when $c = 1$, the frequency of cooperation f_c will decrease as payoff parameter r increases and the threshold r for the presence of clear defectors in the network increases as memory strength c grows, which can be illustrated by fixing $f_c = 0.9$. The r of $c = 0$ $<$ the r of $c = 0.2$ $<$ the r of $c = 0.4$ $<$ the r of $c = 0.6$ $<$ the r of $c = 0.8$. Although defectors will appear in the network, the frequency of cooperation can be enhanced by increasing the memory strength when the payoff parameter is not very



(a) WS (b) RE

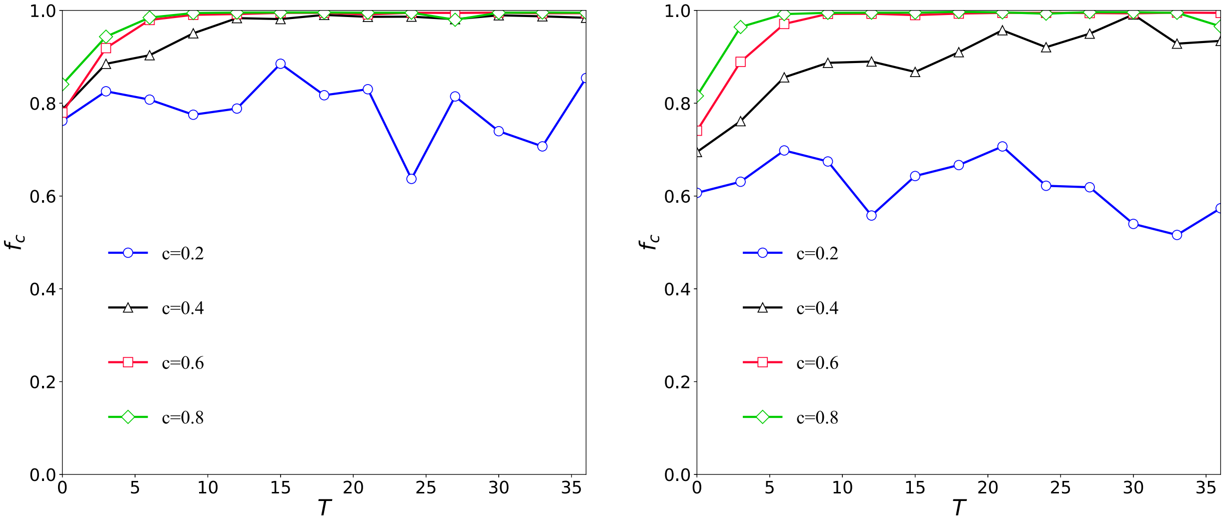
Fig. 4. Plots of cooperation frequency against memory strength. WS is generated by regular graphs and randomized reconnection, where the parameters are set as the number of players $N = 1024$, the random reconnection probability $p = 0.15$, and $K = 4$, which indicates that each player is connected to 2 players (his/her left and right neighbors). RE is generated by a square lattice network with 32 × 32 players. We set the payoff parameter $r = 0.4, 0.6$, and 0.8, and the memory length T is restricted to 5 in both subplots. Setting the evolution time $t = 10^4$, we obtain each cooperation frequency by averaging the cooperators proportion in the last 1000 rounds, and each data point is averaged using the cooperation frequencies of 10 independent simulations. Subplots (a) and (b) show the cooperation frequency against memory strength under different payoff parameters on the WS network and regular network, respectively, where the x-axis and y-axis are set as the memory strength and the cooperation frequency, respectively. Each x-axis range is set as [0, 0.9], and each y-axis range is set as [0, 1]. We observe each network until $t = 10^4$ so that the cooperation frequency becomes stable. For $r = 0.8$, a slow overall decreasing tendency of cooperation frequency with increasing c is shown on both networks. For $r = 0.4$ and $r = 0.6$, both networks show that a larger value of c leads to a larger cooperation frequency.

large ($r < 0.6$). However, the phenomenon will be reversed when $r \geq 0.8$, in which the frequency of cooperation will be inhibited upon increasing the memory strength. There is a difference between Figs. 3(a) and 3(b), where for $c = 0$, players are memoryless and the threshold for the emergence of defectors is apparently higher in the WS network than that in the RE network. Subsequently, we will give a separate explanation for $c = 1$. When $c = 1$, the probability p in Eqs. (3) and (5) that imitate neighbor player y's strategy or alter current strategy will change to $p = \frac{\sum_{i=1}^{T} \delta(S_{x,T}(i), s_y)}{T}$, which means that there is no difference between the profiteers and conformists and they no longer consider the difference in payoffs or the number of strategies among their neighbors. Furthermore, they are only concerned about the strategy they have used in the past T rounds at this point (i.e., the player's strategy will remain the same throughout the evolutionary process). Because the player's strategy at the initial moment is randomly selected from the state space S = {C, D}, the cooperation frequency will gently fluctuate around 0.5 as the payoff parameter changes.

Next, to better discover the relationship between cooperation frequency and memory strength, we show the influence of memory strength on the cooperation frequency under different payoff parameters in Fig. 4, where the x-axis is set as c and the y-axis is set as f_c. Each cooperation frequency is obtained by averaging the cooperation proportion over the last 1000 rounds (we observe each cooperation density until $t = 10^4$ so that the proportion of cooperators in the network becomes stable), and each data point is averaged using the cooperation frequencies of 10 independent simulations. In both Figs. 4(a) and 4(b), the frequency of cooperation is enhanced by increasing c and eventually stabilizes near 1 when $r = 0.4$ and $r = 0.6$. For $r = 0.8$, there is an overall tendency for the frequency of cooperation to be impeded as c increases but cooperators are not extinct and a few cooperators remain in the network. The frequency of cooperation on the WS network has always been higher than that on the RE network. The aforementioned results coincide with the results presented in Fig. 3, which validates our model from another perspective.

3.4. Effect of memory strength and memory length on the cooperation frequency

In this simulation, we focus on the relationship between the cooperation frequency and the memory length. By fixing the payoff parameter $r = 0.6$, setting the x-axis as the memory length T and y-axis as the cooperation frequency f_c, we present the function of cooperation frequency and memory length $T \in [0, 36]$ in Fig. 5. We observe each cooperation density until $t = 10^4$, where each cooperation frequency is obtained by averaging the cooperators proportion over the last 1000 rounds, and each data point is averaged using the cooperation frequencies of over 10 independent realizations. For $c = 0$, the probability p in Eqs. (3) and (5) that imitate neighbor player y's strategy or alter current strategy will



(a) WS (b) RE

Fig. 5. Plots of cooperation frequency against memory length. WS is generated by regular graphs and randomized reconnection, where the parameters are set as the number of players $N = 1024$, the random reconnection probability $p = 0.15$, and $K = 4$, which indicates that each player is connected to two players (his/her left and right neighbors). RE is generated by a square-lattice network with 32×32 players. We set the memory strength $c = 0.2, 0.4, 0.6$, and 0.8, and the payoff parameter r is restricted to 0.6 in both subplots. Setting the evolution time $t = 10^4$, we obtain each cooperation frequency by averaging the cooperators proportion over the last 1000 rounds and each data point is averaged using the cooperation frequencies of 10 independent simulations. Subplots (a) and (b) show the cooperation frequency against memory length under different memory strengths on the WS network and regular network, respectively, where the x-axis and y-axis are set as the memory length and the cooperation frequency, respectively. Each x-axis range is set as [0, 36], and each y-axis range is set as [0, 1]. We observe each network until $t = 10^4$ so that the cooperation frequency becomes stable. For the larger c, the cooperation frequency can be enhanced by increasing memory length, which can be demonstrated from both subplots (a) and (b).

become $p = \frac{1}{1+e^{(\overline{\Pi}_x - \overline{\Pi}_y)/|d_x - d_y|}}$ and $p = \frac{1}{1+e^{(N_{s_x} - k_h)/|d_x - d_y|}}$, which are independent of T. For $c = 1$, the aforementioned equation will turn into $p = \frac{\sum_{i=1}^{T} \delta(S_{x,T}(i), s_y)}{T}$ and the situation we discussed before will occur again, namely, the player's strategy will remain constant during the evolution process. For that reason, we do not set c to 0 or 1, which would become meaningless; however, it is set to 0.2, 0.4, 0.6, and 0.8 instead.

Based on the relationship between the cooperation frequency and the memory length on the WS network, as is shown in Fig. 5(a), the degree of fluctuation of f_c is large by changing T when c is small, which can be seen from $c = 0.2$ marked by blue circles. Moreover, f_c will be improved with the increasing T and finally reach a stable value when c is not too small, which can be seen from $c = 0.4$ marked by black triangles, $c = 0.6$ marked by red squares, and $c = 0.8$ marked by green diamonds. In addition, the f_c corresponding to $c = 0.8$ and $c = 0.6$ keeps increasing as T increases and the f_c corresponding to $c = 0.8$ is always higher than that corresponding to $c = 0.6$ when $T < 15$. However, both f_c curves reach a stable value when $T > 15$ and they are almost identical. In other words, we can eliminate differences in the effect of different c on f_c by increasing T.

Then, we present the relationship between cooperation frequency and memory length on the RE network in Fig. 5(b), where we can obtain similar results to those on the WS network for $c = 0.6$ and $c = 0.8$, which can be stated as f_c growing with T and then reaching a stable value. For $c = 0.4$, although the general trend shows that f_c is increasing with T, it does not reach a stable value and there are fluctuations in the curve. Moreover, the curve for $c = 0.2$ has large fluctuations, similar to the WS network. Furthermore, comparing Fig. 5(a) with Fig. 5(b), we can see that the frequency of cooperation is higher on the WS network for $c = 0.2$ and $c = 0.4$ than that on the RE network, which can indicate that the SG on the WS network promotes cooperation more than that on the RE network in the aforementioned case.

3.5. The evolutionary snapshots of cooperator and defector

To further discuss the memory effect on the cooperation frequency in the square-lattice network, snapshots of cooperators and defectors are given in this section. As shown in Fig. 6, there are snapshots of cooperators and defectors with three pairs of memory strength c and payoff parameter r, which are $(c, r) = (0.3, 0.4)$, $(c, r) = (0.3, 0.6)$, and $(c, r) = (0.5, 0.6)$, with different last and first numbers for contrast and the memory length T is fixed 5. Fig. 6 visualizes the evolutionary game in the network and is more understandable. In addition, to explain the frequency of cooperation with different parameter pairs (c, r) intuitively, snapshots show how cooperators emerge with time when the memory strength c and payoff parameter r change. In detail, the smaller r indicates a higher the cooperation frequency at the

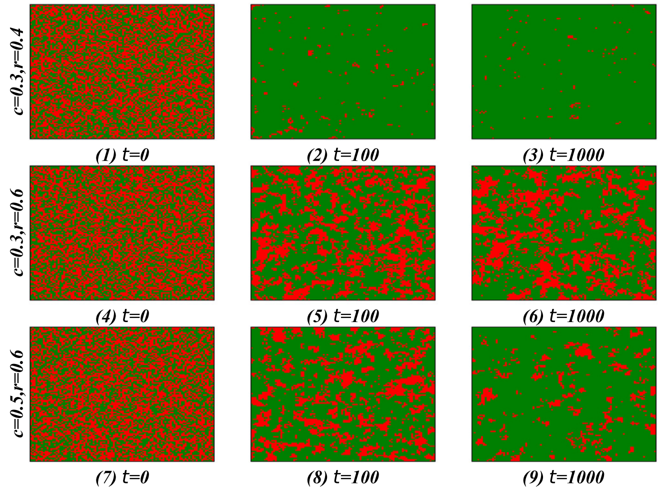


Fig. 6. Characteristic snapshots for different parameter pairs (c, r). Snapshots of equilibrium of cooperators (green) and defectors (red) with three typical combinations of memory strength c, payoff parameter r, and a fixed memory length $T = 5$. The square-lattice network size is set to $N = 100 \times 100$. 50% of cooperators and defectors randomly distribute on networks in the initial state. Each site corresponds to a player. From top to bottom, the parameter pairs (c, r) are set as (0.3, 0.4), (0.3, 0.6), and (0.5, 0.6), respectively. From left to right the time step is fixed to be t = 0, 100, and 1000. The snapshots on the square-lattice network show the emergence of a proliferation of cooperators, which is more intuitive and understandable.

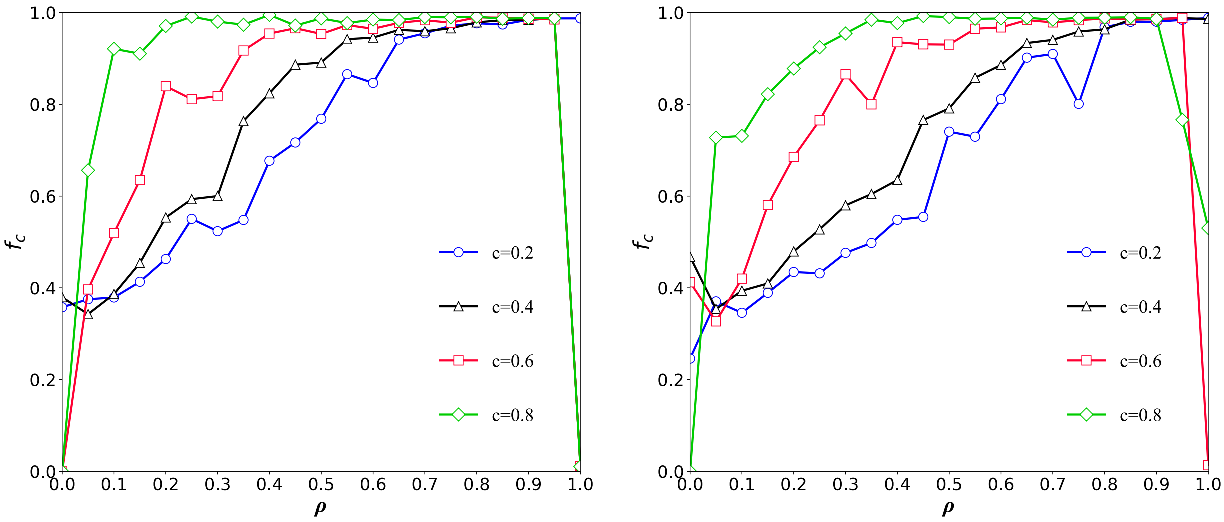
same time with the same c, as evidenced in Fig. 6(2) and Fig. 6(5), Fig. 6(3) and Fig. 6(6). Moreover, when c increases with the same r, a higher c shows a higher cooperation level in the network at the same time, which can be clearly seen from Fig. 6(6) and Fig. 6(9). Furthermore, for each set of parameter pairs (c, r), looking from left to right, we can find that cooperators emerge and prosper by a number of clusters as time progresses.

3.6. Effect of conformist ratio on the cooperation frequency under different memory strengths

In the above studies, the initial type of player is randomly selected from the space T = {P, C}, such that the ratio of conformists is almost equal to 0.5. We will next investigate the effect of different ratios of conformists on network cooperation behavior, where the ratio of conformists in the network is denoted as ρ. By setting the x-axis as ρ and the y-axis as f_c, we present the plots of cooperation frequency against conformist ratio under different memory strengths in Fig. 7. Each cooperation frequency is obtained by averaging the cooperators proportion over the last 1000 rounds, and each data point is averaged using the cooperation frequencies of 10 independent simulations. In Figs. 7(a) and 7(b), we determine that the proportion of cooperation on the WS network increases with increasing numbers of conformists when ρ is less than 0.95, and the proportion of cooperation on the RE network also increases with increasing numbers of conformists when ρ is less than 0.9. A larger c will lead to a smaller ρ, which maximizes the cooperation frequency in the network, indirectly suggesting that memory strength has a positive effect on the emergence of cooperators. In addition, the ratio of cooperators rapidly declines to 0 as the proportion of conformists rises to 0.95 and continues to rise on the WS network when $c \geq 0.4$, and the ratio of cooperators rapidly declines as the proportion of conformists rises to 0.9 and continues to rise on the RE network when $c \geq 0.6$. The proportion of cooperators in the network is approximately 0.5 when $c = 0.8$ and $\rho = 1$, which is different from the WS network. Therefore, the cooperation frequency can be controlled by improving the number of conformists when the ratio of conformists is less than 0.9. The aforementioned results are in agreement with those of the previous studies.

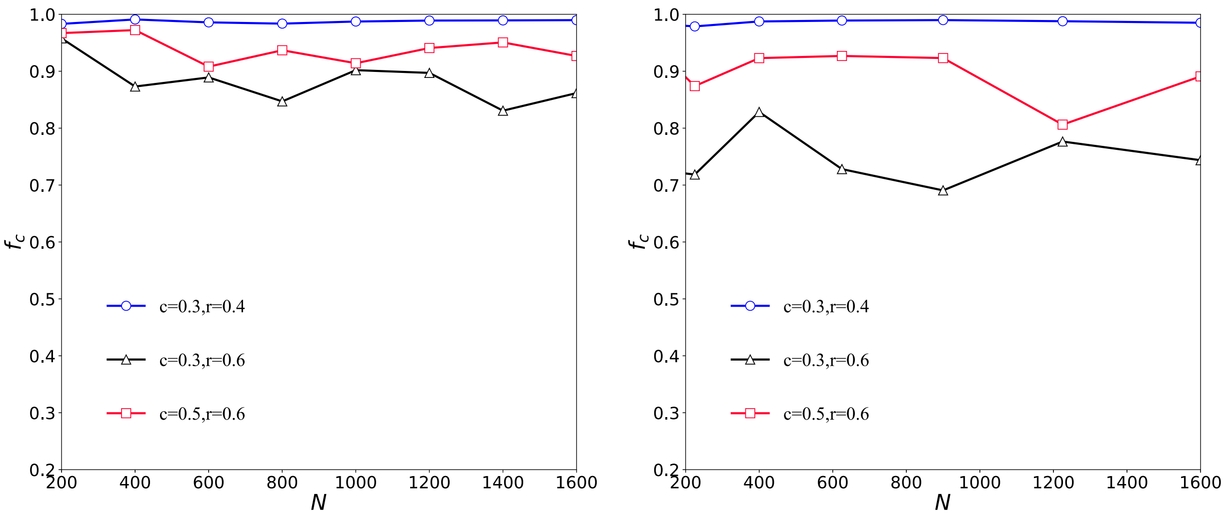
3.7. Effect of network size on the cooperation frequency

Finally, we investigate the effect of network size on the cooperation frequency under different pairs of parameters (c, r) to verify the robustness of our model. We select a memory length of $T = 5$ and three couples of (c, r), and they are (0.3, 0.4), (0.3, 0.6), and (0.5, 0.6), with different last and first numbers for contrast. We observe each cooperation frequency until $t = 5000$, since the frequency of cooperation in the network has reached stability at this time, which can be seen from the simulation results in Fig. 2. From Figs. 8(a) and 8(b), we can obtain that the change in the cooperation frequency with the variation of network size is small for both the WS network and the RE network, while the proportion of cooperators on the RE network fluctuates to a greater extent than that of the WS network. Besides, regardless of network size and network type, with the same $c = 0.3$, the $r = 0.4$ marked by blue circles approaches a higher cooperation frequency and fluctuates less extensively than the $r = 0.6$ marked by black triangles, and with the same $r = 0.6$, the $c = 0.5$ marked by red squares approaches a higher cooperation frequency than the $c = 0.3$ marked by black triangles. In



(a) WS (b) RE

Fig. 7. Plots of cooperation frequency against conformist ratio. WS is generated by regular graphs and randomized reconnection, where the parameters are set as the number of players $N = 1024$, the random reconnection probability $p = 0.15$, and $K = 4$, which indicates that each player is connected to two players (his/her left and right neighbors). RE is generated by a square-lattice network with 32 × 32 players. We set the memory strength $c = 0.2, 0.4, 0.6,$ and 0.8, the payoff parameter r is restricted to 0.6, and the memory length T is restricted to 5 in both subplots. Setting the evolution time $t = 10^4$, we obtain each cooperation frequency by averaging the cooperators proportion over the last 1000 rounds, and each data point is averaged using the cooperation frequencies of 10 independent simulations. Subplots (a) and (b) show the cooperation frequency against conformist ratio under different memory strengths on the WS network and regular network, respectively, where the x-axis and y-axis are set as the conformist ratio and the cooperation frequency, respectively. Each x-axis range is set as [0, 1], and each y-axis range is set as [0, 1]. We observe each network until $t = 10^4$ so that the cooperation frequency becomes stable. The increase of conformists in the network will promote the emergence of cooperation when the ratio of conformists is less than 0.9. Further, the larger c will lead to a smaller ρ, which maximizes the cooperation frequency in the network.



(a) WS (b) RE

Fig. 8. Plots of cooperation frequency against network size. WS is generated by regular graphs and randomized reconnection, where the parameters are set as the random reconnection probability $p = 0.15$, and $K = 4$, which indicates that each player is connected to two players (his/her left and right neighbors). RE is generated by a square-lattice network. The parameter pair (c, r) is set to (0.3, 0.4), (0.3, 0.6), and (0.5, 0.6), with different last and first numbers for contrast, and the memory length T is restricted to 5. Subplots (a) and (b) show the cooperation frequency against network size on the WS network and regular network, respectively, where the x-axis and y-axis are set as the system size and the cooperation frequency, respectively. Each x-axis range is set as [200, 1600], and each y-axis range is set as [0.2, 1]. We observe each network until $t = 5000$ so that the cooperation frequency becomes stable, and each cooperation frequency is obtained by averaging the cooperators proportion over the last 500 rounds. Regardless of the parameter pair (c, r), the cooperation frequency is stable around a certain value as the network size changes.

other words, the higher c and lower r yield a higher f_c, which is consistent with our analysis above. Furthermore, it can be clearly seen that the cooperation frequency is higher on the WS network than on the RE network for the parameter pair $(c, r) = (0.3, 0.6)$. Therefore, we can conclude that the network size has almost no effect on our model, which verifies its excellent robustness.

4. Conclusion and outlook

In this study, we have studied the memory-based SG with profiteers and conformists on networks, including square-lattice networks and small-world networks. Primarily, we give players the property of degree of cooperation, through which we continuousize the payoff when the player plays the SG with his/her neighbors and use it as a measure of similarity between players. Then, we propose two new strategy-updating rules based on profiteers and conformists by incorporating the player's memory mechanism, similarity, and the traditional Fermi strategy-updating rule. In the simulation, the parameter r in the payoff matrix, the conformist ratio ρ, the parameter c, and T in the strategy-updating rules of players, which indicate that the player has a memory mechanism, are investigated to discover their relationships with the final frequency of cooperation f_c, and the robustness of our model is validated. In particular, compared to the player without memory, who has less historical reference information, tends to be shortsighted and chooses a defection strategy in one round that seems to bring greater payoffs, the frequency of cooperation can be enhanced by introducing the memory mechanism. In detail, we can promote cooperation behavior by increasing the memory strength of players. However, this phenomenon will be reversed when r is larger than 0.8, i.e., a smaller memory strength will afford a higher cooperation frequency in this case. In addition, the level of cooperation can be enhanced by improving the player's memory length. This result is both intuitive as well as easily understandable in terms of the real world because people with more vision tend to be more concerned with long-term benefits, while people with narrow vision only care about current benefits, and cooperation is the strategy that can bring the highest returns for the whole network. Furthermore, the implications of different ratios of conformists on network cooperation behavior have been studied, from which we derive that the cooperation frequency can be facilitated by improving the number of conformists when the ratio of conformists is less than 0.9. In the end, we conduct simulations on different sizes of networks and find that network size has almost no effect on the cooperation frequency, which illustrates the robustness of the model we proposed. Therefore, by introducing a memory mechanism, players can become high visionaries, which boost the emergence of cooperation in the network.

In our work, we initially generate the degree of cooperation by making it follow a uniform distribution of [0, 1]. However, different distribution functions, such as the normal distribution, may result in different results. Furthermore, to study the susceptibility of the frequency of cooperation, various game models, including PDGs, public goods games, and SHGs, can be considered. The ratio of conformists we have studied is fixed in the evolutionary process, whereas the conformists and the profiteers learning from each other in the evolutionary process may yield interesting results, which will be the focus of our future work. Furthermore, our strategy-updating rules are a combination of traditional Fermi functions and the combination between different rules of strategy-updating, such as best-take-over, replicator dynamics, and Moran process, may afford new discoveries. Eventually, we hope that our work will contribute to the relative study of evolutionary games with conformists and memory effects in the near future.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Minyu Feng reports financial support was provided by Ministry of Education in China (MOE).

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References

[1] M. Puurtinen, S. Heap, T. Mappes, The joint emergence of group competition and within-group cooperation, Evol. Hum. Behav. 36 (3) (2015) 211–217.
[2] C. Deng, L. Wang, Z. Rong, et al., Cooperation emergence in group population with unequal competitions, Europhys. Lett. 131 (2) (2020) 28001.
[3] M. Nowak, K. Sigmund, A strategy of win-stay, lose-shift that outperforms tit-for-tat in the Prisoner's Dilemma game, Nature 364 (6432) (1993) 56–58.
[4] Z. Wang, M. Perc, Aspiring to the fittest and promotion of cooperation in the prisoner's dilemma game, Phys. Rev. E 82 (2) (2010) 021115.
[5] J. Grujić, C. Fosco, L. Araujo, et al., Social experiments in the mesoscale: Humans playing a spatial prisoner's dilemma, PLoS One 5 (11) (2010) e13749.
[6] C. Hauert, M. Doebeli, Spatial structure often inhibits the evolution of cooperation in the snowdrift game, Nature 428 (6983) (2004) 643–646.
[7] W.B. Du, X.B. Cao, M.B. Hu, et al., Asymmetric cost in snowdrift game on scale-free networks, Europhys. Lett. 87 (6) (2009) 60004.
[8] M. Doebeli, C. Hauert, Models of cooperation based on the Prisoner's Dilemma and the Snowdrift game, Ecol. Lett. 8 (7) (2005) 748–766.
[9] M. Starnini, A. Sánchez, J. Poncela, et al., Coordination and growth: the stag hunt game on evolutionary networks, J. Stat. Mech. Theory Exp. 2011 (05) (2011) P05008.

[10] L. Wang, C. Xia, L. Wang, et al., An evolving stag-hunt game with elimination and reproduction on regular lattices, Chaos Solitons Fractals 56 (2013) 69–76.
[11] J.M. Pacheco, F.C. Santos, M.O. Souza, et al., Evolutionary dynamics of collective action in N-person stag hunt dilemmas, Proc. R. Soc. B: Biol. Sci. 276 (1655) (2009) 315–321.
[12] X. Wang, C. Gu, J. Zhao, et al., Evolutionary game dynamics of combining the imitation and aspiration-driven update rules, Phys. Rev. E 100 (2) (2019) 022411.
[13] Y. Fang, W. Wei, S. Mei, et al., Promoting electric vehicle charging infrastructure considering policy incentives and user preferences: An evolutionary game model in a small-world network, J. Cleaner Prod. 258 (2020) 120753.
[14] S.N. Menon, V. Sasidevan, S. Sinha, Emergence of cooperation as a non-equilibrium transition in noisy spatial games, Front. Phys. 6 (2018) 34.
[15] F.C. Santos, J.M. Pacheco, Scale-free networks provide a unifying framework for the emergence of cooperation, Phys. Rev. Lett. 95 (9) (2005) 098104.
[16] H. Xu, C. Tian, W. Ye, et al., Effects of investors' power correlations in the power-based game on networks, Physica A 506 (2018) 424–432.
[17] M. Feng, L.J. Deng, F. Chen, et al., The accumulative law and its probability model: an extension of the Pareto distribution and the log-normal distribution, Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci. 476 (2237) (2020) 20200019.
[18] M. Feng, L. Deng, J. Kurths, Evolving networks based on birth and death process regarding the scale stationarity, Chaos 28 (8) (2018) 083118.
[19] M. Feng, H. Qu, Z. Yi, et al., Evolving scale-free networks by Poisson process: modeling and degree distribution, IEEE Trans. Cybern. 46 (5) (2015) 1144–1155.
[20] M.A. Nowak, Five rules for the evolution of cooperation, Science 314 (5805) (2006) 1560–1563.
[21] J.L. Barker, J.L. Bronstein, M.L. Friesen, et al., Synthesizing perspectives on the evolution of cooperation within and between species, Evolution 71 (4) (2017) 814–825.
[22] R.L. Trivers, The evolution of reciprocal altruism, Q. Rev. Biol. 46 (1) (1971) 35–57.
[23] S. Righi, K. Takács, Social closure and the evolution of cooperation via indirect reciprocity, Sci. Rep. 8 (1) (2018) 1–9.
[24] D.S. Wilson, A theory of group selection, Proc. Natl. Acad. Sci. 72 (1) (1975) 143–146.
[25] X. Li, M. Jusup, Z. Wang, et al., Punishment diminishes the benefits of network reciprocity in social dilemma experiments, Proc. Natl. Acad. Sci. 115 (1) (2018) 30–35.
[26] A. Szolnoki, M. Perc, Conformity enhances network reciprocity in evolutionary social dilemmas, J. R. Soc. Interface 12 (103) (2015) 20141299.
[27] K.M.A. Kabir, J. Tanimoto, Z. Wang, Influence of bolstering network reciprocity in the evolutionary spatial prisoner's dilemma game: A perspective, Eur. Phys. J. B 91 (12) (2018) 1–10.
[28] J. Andreoni, W. Harbaugh, L. Vesterlund, The carrot or the stick: Rewards, punishments, and cooperation, Am. Econ. Rev. 93 (3) (2003) 893–902.
[29] M. Milinski, D. Semmann, H.J. Krambeck, Reputation helps solve the 'tragedy of the commons', Nature 415 (6870) (2002) 424–426.
[30] H.X. Yang, X. Chen, Promoting cooperation by punishing minority, Appl. Math. Comput. 316 (2018) 460–466.
[31] C.W. Huang, Q.L. Dai, Persistence paves the way for cooperation in evolutionary games, Europhys. Lett. 118 (2) (2017) 28002.
[32] A. Szolnoki, M. Perc, Collective influence in evolutionary social dilemmas, Europhys. Lett. 113 (5) (2016) 58004.
[33] J. Qin, Y. Chen, Y. Kang, et al., Social diversity promotes cooperation in spatial multigames, Europhys. Lett. 118 (1) (2017) 18002.
[34] Z. Danku, M. Perc, A. Szolnoki, Knowing the past improves cooperation in the future, Sci. Rep. 9 (1) (2019) 1–9.
[35] V. Capraro, M. Perc, Mathematical foundations of moral preferences, J. R. Soc. Interface 18 (175) (2021) 20200880.
[36] Y. Liu, C. Huang, Q. Dai, Preferential selection based on strategy persistence and memory promotes cooperation in evolutionary prisoner's dilemma games, Physica A 499 (2018) 481–489.
[37] F. Shu, Y. Liu, X. Liu, et al., Memory-based conformity enhances cooperation in social dilemmas, Appl. Math. Comput. 346 (2019) 480–490.
[38] J. Zhao, C. Luo, The effect of preferential teaching and memory on cooperation clusters in interdependent networks, Appl. Math. Comput. 363 (2019) 124575.
[39] Y. Dong, H. Xu, S. Fan, Memory-based stag hunt game on regular lattices, Physica A 519 (2019) 247–255.
[40] W. Lu, J. Wang, C. Xia, Role of memory effect in the evolution of cooperation based on spatial prisoner's dilemma game, Phys. Lett. A 382 (42–43) (2018) 3058–3063.
[41] Z. Niu, J. Xu, D. Dai, et al., Rational conformity behavior can promote cooperation in the prisoner's dilemma game, Chaos Solitons Fractals 112 (2018) 92–96.
[42] Z. Yang, Z. Li, L. Wang, Evolution of cooperation in a conformity-driven evolving dynamic social network, Appl. Math. Comput. 379 (2020) 125251.
[43] L. Zhang, C. Huang, H. Li, et al., Cooperation guided by imitation, aspiration and conformity-driven dynamics in evolutionary games, Phys. A: Stat. Mech. Appl. 561 (2021) 125260.
[44] B. Pi, Z. Zeng, M. Feng, et al., Evolutionary multigame with conformists and profiteers based on dynamic complex networks, Chaos 32 (2) (2022) 023117.
[45] A. Szolnoki, M. Perc, Leaders should not be conformists in evolutionary social dilemmas, Sci. Rep. 6 (1) (2016) 1–8.
[46] F. Shu, X. Liu, K. Fang, et al., Memory-based snowdrift game on a square lattice, Physica A 496 (2018) 15–26.
[47] D.J. Watts, S.H. Strogatz, Collective dynamics of 'small-world' networks, Nature 393 (6684) (1998) 440–442.